Stats 300 **Spring 2024**

Lab Assignment #15

This lab is due at 9:35 AM on Monday, 3/25 and is worth 12 points. This may be done individually, or in a group of 2 or 3 people.

Write each probability answer as a fraction, or a decimal to at least 3 significant digits, or a percent to at least 3 significant digits. Show work.

Part 1: Normal Approximation to Binomial Distribution

1) Suppose that 18.2% of all Dum Dum Pops are Mystery flavor. A bag contains 600 Dum Dum Pops.

a) Verify that the normal approximation to binomial distribution is valid by checking that np and $n(1-p)$ and both greater than 10.

b) Calculate μ and σ for the NABD.

c) What is the probability that there will be at most 105 Mystery flavor in the bag of 600?

d) What is the probability that there will be exactly 99 Mystery flavor?

e) What is the probability that there will be between 100 and 110 Mystery flavor, inclusive.

2) A large study shows that 35% of all drivers say that the biggest driving hazard is drivers following too closely. In a smaller survey, 203 people are surveyed.

a) Verify that the normal approximation to binomial distribution is valid by checking that np and $n(1-p)$ and both greater than 10.

b) Calculate μ and σ for the NABD.

c) What is the probability that more than 75 of the 203 people will say that the biggest driving hazard is drivers following too closely?

d) What is the probability that fewer than 75 people will say that the biggest driving hazard is drivers following too closely?

e) What is the probability that exactly 75 people will say that the biggest driving hazard is drivers following too closely?

f) Do your answers to (c)-(e) add to 100%?

Part 2: Advanced probability questions

1) You and a friend play rock-paper-scissors. Each player has a 50% chance of winning, right?

Sometimes the game is won in the first round. (For example, you play rock and your friend plays scissors.) Sometimes the game is tied in the first round (example: you and your friend both play paper) and goes to a second round.

a) List all the outcomes that can happen in the first round. There are nine, right?

Assume that each of these nine outcomes is equally likely.

b) What's the probability that the game is tied in the first round?

c) What's the probability that you win in the first round?

d) What's the probability that you lose in the first round?

Note: parts (b)-(d) should add to 100%, and none are zero.

e) What's the probability that you win but it takes two rounds? (Note: in order for this to happen, the game must be tied through one round, and then you win in the second round.)

f) What's the probability that you win but it takes three rounds? (Note: in order for this to happen, the game must be tied through one round, and then tied in the second round, and then you win in the third round.)

g) What's the probability that you win but it takes four rounds? (Note: in order for this to happen, the game must be tied through one round, and then tied in the second round, and then tied in the third round, and then you win in the fourth round.)

You can win rock-paper-scissors by winning in the first round OR the second round OR the third round OR ... (any round thereafter)

h) Write an infinite sum of fractions representing the probability that you win.

Definition: A geometric sequence is a list of numbers. The ratio of every pair of consecutive numbers is equal.
Example: 3, 6, 12, 24, 48
Example: 4, 0.4, 0.04, 0.004, 0.0004, …

A geometric sequence can be finite (it ends) or infinite (it does not end).

Definition: A geometric series is the sum of numbers in a geometric sequence.
Example: 4 + 0.4 + 0.04 + 0.004 + 0.0004 + …

Weird fact: An infinite geometric series has a final sum, assuming that the ratio of terms (ratio of each number in the series divided by the previous number) is smaller than 1. Even though an infinite number of numbers are added together.

Ex: $4 + 0.4 + 0.04 + 0.004 + 0.0004 + \ldots = 4.4444\ldots = 4\frac{4}{9}$.

Good thing, because this allows us to calculate probabilities of games that potentially have an unlimited number of rounds.

The formula for the sum (S) of an infinite geometric series is:

$$S = \frac{a}{1-r},$$

where a is the first number in the series, and r is the ratio of each number in the series divided by the previous number.

Ex: in the series just above, $a = 4$, $r = 1/10$ (0.4 divided by 4 equals 1/10),
so $S = 4/(1 - 1/10) = 4/(9/10) = 40/9 = 4\frac{4}{9}$.

i) For the series you wrote in (h), what is the first term? What is the ratio of terms?

j) For the series you wrote in (h), what is the sum? Is this the answer you expected? (See the second sentence of the problem.)

2) You and a friend play rock-paper-scissors-lizard-Spock. (See Wikipedia or elsewhere.) Each player has a 50% chance of winning, right?

Sometimes the game is won in the first round. (For example, you play paper and your friend plays Spock.) Sometimes the game is tied in the first round (example: you and your friend both play lizard) and goes to a second round.

a) List all the outcomes that can happen in the first round. There are 25, right?

Assume that each of these 25 outcomes is equally likely.

b) What's the probability that the game is tied in the first round?

c) What's the probability that you win in the first round?

d) What's the probability that you lose in the first round?

Note: parts (b)-(d) should add to 100%, and none are zero, and these are not the same numbers as in problem 1.

e) What's the probability that you win but it takes two rounds? (Note: in order for this to happen, the game must be tied through one round, and then you win in the second round.)

f) What's the probability that you win but it takes three rounds? (Note: in order for this to happen, the game must be tied through one round, and then tied in the second round, and then you win in the third round.)

g) What's the probability that you win but it takes four rounds? (Note: in order for this to happen, the game must be tied through one round, and then tied in the second round, and then tied in the third round, and then you win in the fourth round.)

h) Write an infinite sum of fractions representing the probability that you win.

i) For the series you wrote in (h), what is the first term? What is the ratio of terms?

j) For the series you wrote in (h), what is the sum? Is this the answer you expected? (See the second sentence of the problem.)

3) All job applicants at the Crunch Bunnies factory have to take a drug test before being hired. The test is 99% accurate for negative results. This means that if a person is not using drugs, there is a 99% chance that the test will be correct. The test is 95% accurate for positive results. This means that if a person is using drugs, there is a 95% chance that the test will be correct.

a) If 3% of applicants are using drugs, what percent of positive test results are correct? Hint: make a tree diagram, starting with the 3%. Hint: another option is to assume that there are 10000 applicants. Note: the answer to part (a) is not 2.85%. The answer to part (a) is not 3.82% either.

b) If 30% of applicants are using drugs, what percent of positive test results are correct? Note: the answer to part (b) is not 28.5%. And it is not 29.2% either.

c) For the purpose of getting a larger percentage of correct positive test results, what's better: fewer drug users (part a) or more drug users (part b)?