Statistics 300 Name: Solution **Spring 2009**

Instructor: Lawrence C. Larsen **Tue./Thu. 7:00 - 9:05 p.m.**

Exam #2

(8 points : 8 minutes)

1. **X is a random variable and X ~ N(μ = 80,σ = 15).**
   A random sample of values from another "normally distributed " population (y) has the statistics shown in the box below. Test the idea that the sample comes from a population with variabilty less than the variability of X. about variation, so σ, not μ not p
   For this test (not a confid. interval), use a 0.05 significance level.

Sample statistics
$\bar{y}$ = 39.2
s_y = 11.8
n = 28

df = 27

idea $\sigma < 15$ the variability of X

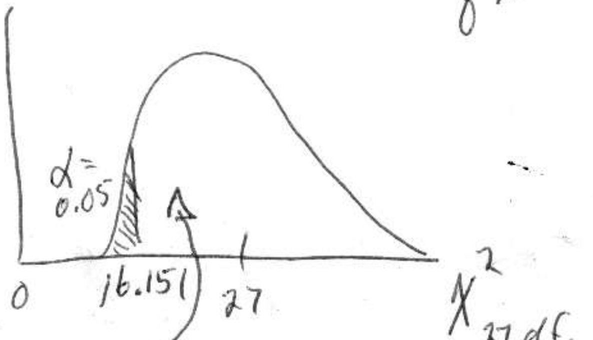
H₀: $\sigma \geq 15$

H₁: $\sigma < 15$

$\alpha = 0.05$ left tail

Test Statistic

$$\frac{(n-1)s^2}{\sigma_0^2} = \frac{(28-1)(11.8)^2}{(15)^2} = \boxed{16.709}$$



Do Not reject Ho!

(7 points : 7 minutes)

2. **Consumer reports followed the experience of 568 randomly selected people who bought Yugo cars. Major engine repairs were needed on 96 of the cars during the 3 year or 30,000 mile warranty period. Use these results to make a 94% confidence interval for the proportion of all Yugo cars that needed major engine repairs during the warranty period.**

$$94\%\ CI(p) = \hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} = 0.169 \pm 1.88\sqrt{\frac{(0.169)(0.831)}{568}}$$

$$= 0.169 \pm 0.030$$

$$= [0.139 < p < 0.199]$$

$\hat{p} = 96/568 = 0.169$

$\hat{q} = 1 - \hat{p} = 0.831$

$n = 568$

confid = 0.94

$\alpha = 0.06$

$\alpha/2 = 0.03$ $z_{\alpha/2} = 1.88$

Based on your results, was it reasonable for the Yugo company to claim that less than 20% of all Yugos would need major engine repairs during the warranty period.
Circle "Yes" or "No" and say why.

YES **NO** **Why:** Because the CI contains values (whole interval) that are < 20%.

(6 points : 6 minutes)

3. When testing poisons on ants, the dose that kills 50% of the ants is called the LD50 (lethal dose 50%). When testing the % killed by different doses, a company wants to be 95% sure that the proportion of ants killed in each of their samples is within one percentage point of the true percent of all ants that would die at that dose. Help plan the company's experiments by deciding how many ants they need to test for a dose near the LD50.

sample size to estimate p

$$n = \frac{(z_{\alpha/2})^2 \hat{p}\hat{q}}{E^2}$$

$$= \frac{(1.96)^2(.5)(.5)}{(0.01)^2}$$

$$= 9604$$

$E = 0.01$

$\hat{p} = 0.50$ ← near LD50

$\hat{q} = 0.50$

confidence = 95%

$z_{\alpha/2} = 1.96$

(6 points : 6 minutes)

4. To determine the number of calories in a so-called "serving" of cookies, a laboratory uses a method of measurement that has variability characterized by a standard deviation of 0.65 calories. If the laboratory must estimate the true mean calorie content of all "servings" of these cookies within 0.1 calories with 90% confidence, how many servings must they test?

$\hat{\sigma} = 0.65$ $E = 0.1$ $\alpha = 0.10$ $\alpha/2 = 0.05$ $z_{\alpha/2} = 1.645$

sample size for estimating μ

$$n = \left[\frac{(z_{\alpha/2})(\hat{\sigma})}{(E)}\right]^2$$

$$= \left[\frac{(1.645)(0.65)}{(0.1)}\right]^2$$

$$= 114.33 \uparrow 115$$

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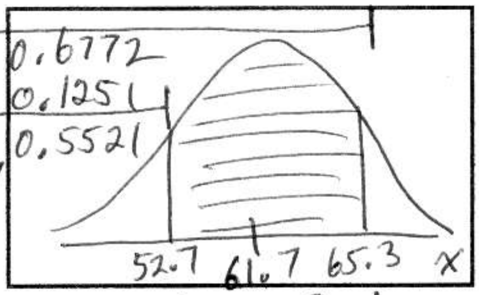
(5 points : 4 minutes)

5. **Given: X has a Normal distribution with $\mu = 61.7$ and $\sigma = 7.8$. What is the probability that a random value of X will be between 52.7 and 65.3?**

Draw the picture for the problem and calculate the probability.



answer = 0.5521

$\sigma = 7.8$

$Z_1 = \frac{52.7-61.7}{7.8} = -1.15$ 0 $Z_2 = \frac{65.3-61.7}{7.8} = 0.46$

Z	.05
-1.1	$\boxed{.1251}$

Z	.06
0.4	$\boxed{.6772}$

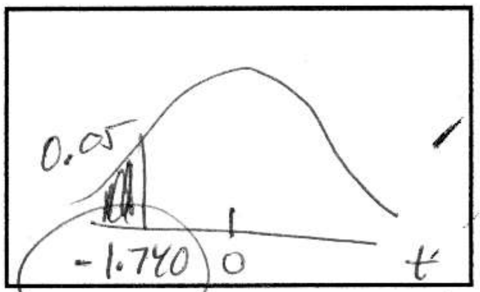
(3 points : 3 minutes)

6. **What is the 5th percentile of the "t" distribution with 17 degrees of freedom?**

Draw the picture for the problem and determine the value of the percentile.

Table A.3 t distribution

df	prob in one tail = .05
17	$\boxed{1.740}$



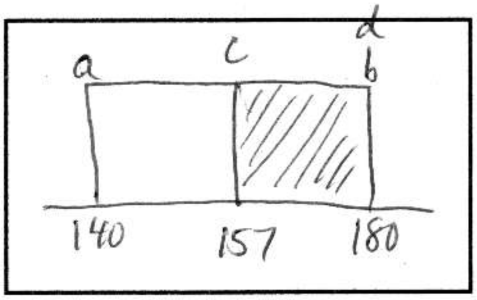
17 df.

$-1.740 = P_5$

(4 points : 3 minutes)

7. **A random variable, X, has a Uniform distribution on the interval [140,180]. What is the probability that a random value from this distribution will be greater than 157?**

Draw the picture for the problem and calculate the probability.



$$P(c < x < d) = \frac{d-c}{b-a}$$

$$= \frac{180-157}{180-140}$$

$$= 23/40 = 0.575$$

(8 points : 8 minutes)

8. **An engineer tests materials for strength. Force is applied to each item until it breaks. A random sample of 14 items is tested and the force needed to break each one is recorded. The breaking forces averaged 3.465 tons with a standard deviation of 0.122 tons. A histogram of the breaking forces is shown below. Use the data to construct a 95% confidence interval for the true mean breaking force for the whole population of items from which the sample was selected.**

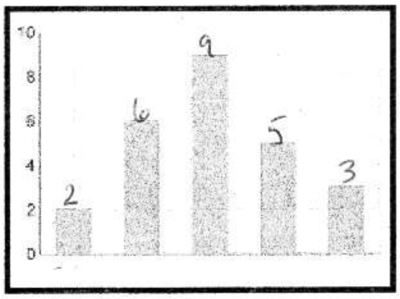
$$95\%\ CI(\mu) = \bar{x} \pm t\left(\frac{s}{\sqrt{n}}\right)$$

$$= 3.465 \pm 2.160\left(\frac{0.122}{\sqrt{14}}\right)$$

$$= 3.465 \pm 0.070$$

$$[3.395 < \mu < 3.535]$$

$\bar{x} = 3.465$
$s = 0.122$
$n = 14$
$df = 13$
$\alpha = 0.05$ 2 tails
$t = 2.160$



Based on your confidence interval, is it reasonable for the engineer to claim that the population of materials satisfies the goal of μ > 3.7 tons breaking strength?

YES NO Why? Because none of the values in the CI, the reasonable range, are > 3.7 tons.

(7 points : 7 minutes)

9. **A company supplies a specific part to a client that makes high quality refrigerators. All of the parts must be very uniform in length to be acceptable. A random sample of 23 parts is selected and the length of each one is measured. The results are summarized below. Use these results to prepare a 95% confidence interval for the standard deviation of the lengths of all the parts that the company manufactures for their client.** (The distribution of lengths is rather bell-shaped.)

Results from sample
4.002 cm = $\bar{x}$
0.004 cm = s
23 = n

$22 = df.$

$\alpha = 0.05$

$\alpha/2 = 0.025$ in each tail

$\chi^2_L = 10.982$

$\chi^2_R = 36.781$

95% CI(σ) :

$$\sqrt{\frac{(n-1)s^2}{\chi^2_R}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi^2_L}}$$

$$\sqrt{\frac{(23-1)(0.004)^2}{36.781}} < \sigma < \sqrt{\frac{(23-1)(0.004)^2}{10.982}}$$

$$[0.0031 < \sigma < 0.0057]$$

(6 points : 6 minutes)

10. The birthweights of all babies born in the USA last year had an average of 3.08 kg and a standard deviation of 0.62 kg. If a random set of 15 babies is selected from those born last year in the USA, what is the probability that the average weight of the ~~17~~ 15 babies will be greater than 3.15 kg? (The distribution of birth weights is known to be "bell-shaped".)

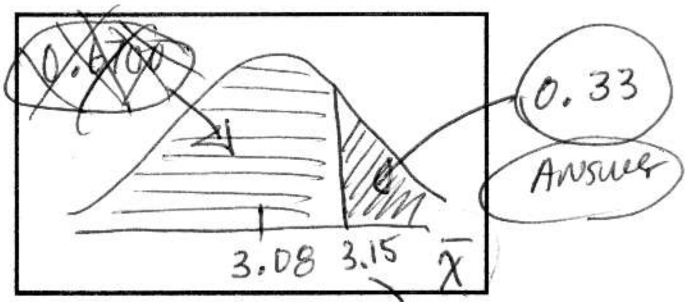
Picture is worth two of the 6 points!

$X \sim N(3.08 = \mu, 0.62 = \sigma)$

$n = 15$

$\bar{X} \sim N\left(3.08 = \mu_{\bar{x}}, \frac{0.62}{\sqrt{15}} = \sigma_{\bar{x}}\right)$

$= 0.16$



$\sigma_{\bar{x}} = 0.16$

0 Z

$\frac{3.15 - 3.08}{0.16} = 0.44$

.04

0.4 — $\boxed{0.6700}$

~~answer~~

(5 points : 6 minutes)

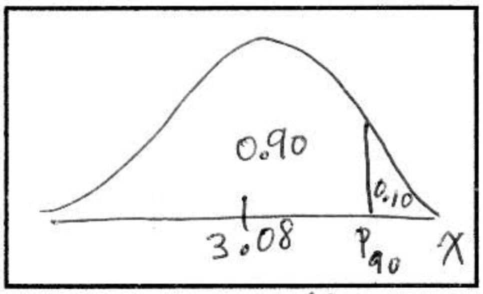
11. The birthweights of all babies born in the USA last year had an average of 3.08 kg and a standard deviation of 0.62 kg. If these weights were Normally distributed, what was the 90th percentile of those weights?

Picture is worth two of the 5 points!

$\frac{P_{90} - 3.08}{0.62} = 1.28 = Z_{90}$

$P_{90} = (1.28)(0.62) + 3.08$

$= 3.87$



$\sigma = 0.62$

$Z_{90} = 1.28$

.08

1.2 — $\boxed{.8997}$

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(8 points)

12. An auto insurance company advertises that 95% of its customers are satisfied with the company's products and services. A random sample of the company's customers included 770 that were satisfied and 60 that were not satisfied. Use these results to test whether the company's advertized percentage is too high. (Use a 2% significance level for your test.)

true $\rightarrow$ advertized p

Claim : $p < 0.95$

H₀: $p \geq 0.95$

H₁: $p < 0.95$

$\alpha = 0.02$ left tail

Satisfied = 770

Not = 60

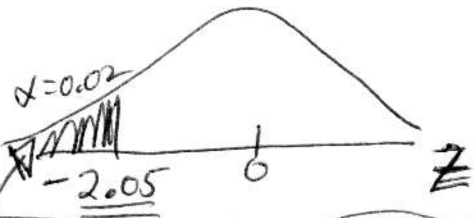
Total = 830 = N

$0.9277 = \frac{770}{830} = \hat{p}$

$0.0723 = \hat{q}$

Test Statistic

$$\frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{0.9277 - 0.95}{\sqrt{\frac{(.95)(.05)}{830}}} = \frac{-0.0223}{.00756} = -2.95$$



If the null hypothesis (H₀:) is true, what is the probability of a "Type I" error for this test? P(Type I error) = 0.02

= α

= significance level

Reject H₀!

(8 points : 12 minutes)

13. The form (tablets or liquid) of a sleeping aid medication may affect the speed at which the medication works. Use the results of the experiment below to test the claim that the average amount of time (elapsed time) before patients fall asleep is at least 2 minutes longer for tablets than for liquid. It is interesting but not especially important that the variation in time was about the same for both tablets and liquid. The seven patients who participated in the study did not like the flavor of the liquid.
(Let $\alpha = 0.05$ for this test.)

claim: $\mu_T \geq \mu_L + 2$

$\mu_d = (\mu_T - \mu_L) \geq 2$ (order is T − L)

H₀: $\mu_d \geq 2$

H₁: $\mu_d < 2$

$\alpha = 0.05$ left tail

matched pairs

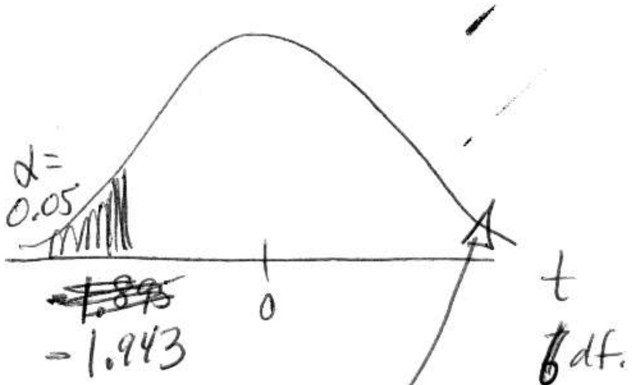
Minutes Before Sleep for			(T−L)
Patient	Tablet	Liquid	d
1	30	25	5
2	18	14	4
3	29	23	6
4	19	14	5
5	24	25	−1
6	23	17	6
7	21	16	5
$\bar{x}$ =	22.33	18.17	$4.286 = \bar{d}$
s =	3.98	4.71	$2.430 = s_d$
n =	7	7	$7 = n$

df = 6

Test Statistic

$$\frac{\bar{d} - \mu_d}{s_d/\sqrt{n}} = \frac{4.286 - 2}{2.430/\sqrt{7}}$$

$$= \frac{2.286}{0.918} = 2.490$$



Do Not reject H₀;

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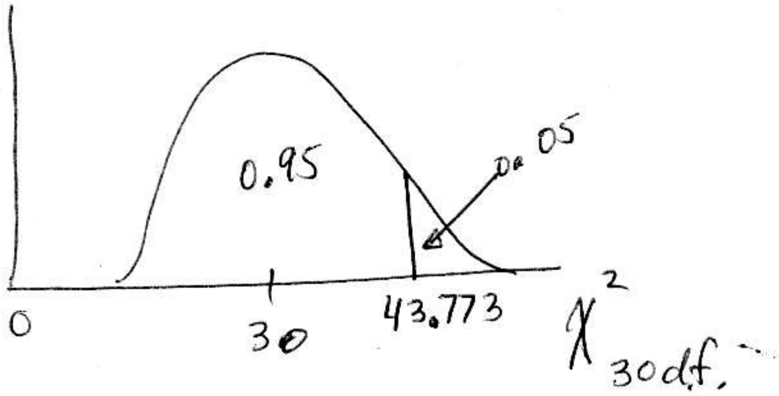
(5 points)
Extra Credit Problem

A random variable (call it "X") has a Chisquare distribution with 30 degrees of freedom. In a random sample of 10 values of X, what is the probability that at least one of the 10 will be above 43.773 ?



$$P(x < 43.773) = 0.95$$

$$P(x > 43.773) = 0.05$$

$$P(\text{at least one of 10 } x \text{ values will be} > 43.773) =$$

$$1 - P(\text{all are} < 43.773) =$$

$$1 - (0.95)^{10} = 0.401$$

answer