Statistics 300
Quiz #10

Name: Solution

In these problems, you must provide the symbolic formula and the formula with the relevant values in place.

(8 points)

1. A survey of Sacramento Area households found that 285 had a dog (one or more) and 595 did not. Use these results to prepare a 95% confidence interval for the proportion of all households in the Sacramento Area that have a dog.

$n = 285 + 595 = 880$

$\hat{p} = 285/880 = 0.324$

$\hat{q} = 1 - \hat{p} = 0.676$

$\alpha = 1 - \text{confid} = 1 - 0.95 = 0.05$

$\alpha/2 = 0.025$

$Z_{\alpha/2} = 1.96$

$95\%\ CI(p) = \hat{p} \pm Z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$

$= 0.324 \pm 1.96\sqrt{\frac{(.324)(.676)}{880}}$

$= 0.324 \pm 0.031$

$[0.293 < p < 0.355]$

Does the interval you prepared above make it reasonable to claim that 2 out of every 5 households in the Sacramento Area have a dog?

Yes **No** **Why?** Because 0.40 or 40% is not in the CI(p), which is the reasonable range for the true p.

$2/5 = 0.40$

(6 points)

2. The Board of Supervisors for Sacramento County wants to know theproportion of all households In the area that have a dog (one or more). If they want to be 90% certain that the difference between the proportion in their sample and the proportion for the whole county is not more than 0.05, how many households must they sample at random? In 1950, it was estimated that 60% of Sacramento area households had a dog.

sample size for estimating p

$\alpha = 1 - \text{confidence}$

$= 1 - 0.90 = 0.10$

$\alpha/2 = 0.05$

$Z_{\alpha/2} = 1.645$

$\hat{p} = 0.60$ [from 1950 study]

$\hat{q} = 1 - \hat{p} = 0.40$

$E = 0.05$ [acceptable difference $|p - \hat{p}|$].

$n = \frac{(Z_{\alpha/2})^2 \hat{p}\hat{q}}{E^2}$

$= \frac{(1.645)^2(0.6)(0.4)}{(0.05)^2}$

$= 259.8 \uparrow 260$

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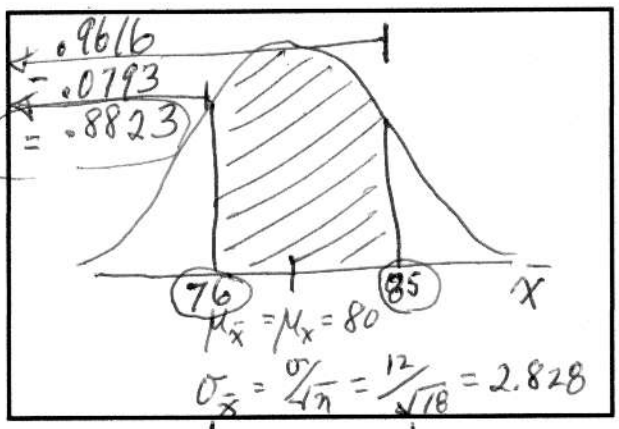
(6 points)

3. Shade in the area that corresponds to the probability statement, then determine the probability (picture is worth 2 points).

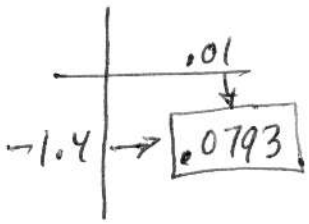


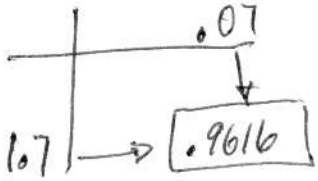
$\frac{76-80}{2.828} = -1.41$ $\quad$ $\frac{85-80}{2.828} = 1.77$

$X \sim N(\mu = 80, \sigma = 12)$

What is the probability that the average for a random sample of 18 values will be between 76 and 85?

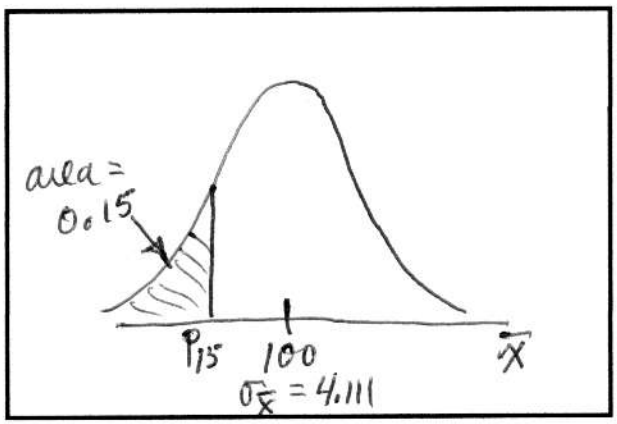
$P(76 < \bar{x} < 85) =$ 0.8823





(6 points)

4. Shade in the area that corresponds to the probability statement, then determine the required answer (picture is worth 2 points).



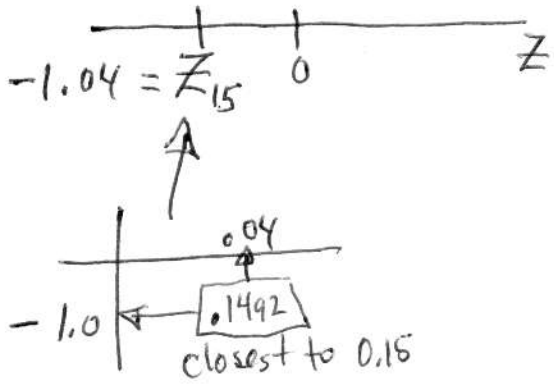
$X \sim N(\mu = 100, \sigma = 13)$

What is the 15th percentile of the population of all possible sample means from samples of size = 10?

$\mu_{\bar{x}} = \mu_x = 100$

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{13}{\sqrt{10}} = 4.111$

$P_{15} =$ 95.72



$\frac{P_{15} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{P_{15} - 100}{4.111} = Z_{15} = -1.04$

$P_{15} = (-1.04)(4.111) + 100 = 95.72$

$= (Z_{15})(\sigma_{\bar{x}}) + \mu_{\bar{x}}$