Solutions

More counting problems.

1. **You are part of a group of 13 people waiting to buy tickets for the next movie about the X-Men. The theater manager decides that you will all form one line with the order picked at random. Of course, that means that all possible ways the line can be formed are equally likely to be the final order. How many ways can the line be formed, and what is the probability that you will find yourself at the front of the line?**

The 13 people are distinct human beings.
Each different way the line can be formed is a permutation.
So, $n = 13$ $n! = 13! = 6,227,020,800$ OR 6.2×10^9

or ${}_{13}P_{13} = 6,227,020,800$ OR 6.2×10^9

same answer

2. **You are travelling by airplane to another city. Before you boarded the airplane, you checked six (6) different pieces of luggage. Sadly, two of the six pieces of luggage will be lost by the airline. The six pieces of luggage are (1) a hat box, (2) a large suitcase, (3) a small suitcase, (4) a duffle bag, (5) a box of books, and (6) a makeup case. How many different sets of four luggage pieces are possible for you to have at the end of your trip.**

The set is the same regardless of the order in which they arrive at the baggage claim. Order does not matter, and the items are distinct.

${}_6C_4 = 15$ different sets of 4 items are possible.

3. **For a new "code" used by a network of spies, every word in the English language will be turned into a unique set of six letters. For example, the word "hot" might be coded as "xoopsp". As you can see, each letter can be used more than once. How many different words could be coded in this way, given that there are 26 letters in the English alphabet?**

six letters will be used _ _ _ _ _ _.
Since each letter can be used more than once, the solution is $26 \cdot 26 \cdot 26 \cdot 26 \cdot 26 \cdot 26$.

$= (26)^6 = 308,915,776$

Solutions

More counting problems (continued).

4. **A row of tacos includes 3 plain, 4 with mild sauce, 5 with hot sauce, and 6 with "fire" sauce. Five of the 18 tacos will be picked at random. What is the probability that the five selected tacos will be "mild, mild, fire, hot, fire" in that order?** (show how you got your answer)

3 P
4 M
5 H
6 F
} 18 tacos

$P(M, M, F, H, F) = \left(\frac{4}{18}\right)\left(\frac{3}{17}\right)\left(\frac{6}{16}\right)\left(\frac{5}{15}\right)\left(\frac{5}{14}\right)$

$= 0.00175$

5. **A local Pet store has 7 cats and 4 dogs waiting to be adopted. Each animal is distinct from all of the others. Tomorrow, 3 people will come to the store and adopt one of these pets. How many possible ways could the 3 adoptions occur?** (Example: Dog3, Cat6, Cat2)

11 animals, each distinct from the others. Order does make a difference, so the answer is $_{11}P_3 = 990$ ways.

If $D_1 D_2 D_4$ would be the same as $D_1 D_4 D_2$, the answer would be $\frac{11!}{7!\,4!} = 330$ [But, this would be a different problem].

6. **A small rural town has 116 people. The safety of the town's water supply is suspect because mercury was used in gold mining 120 years ago. Blood tests will be done on 30 people from the town to see if mercury is present at unsafe levels in their blood. How many different groups of 30 people are possible for the study designers to choose?**

the group is the same regardless of how the people are arranged, so order does not make a difference.

$_{116}C_{30} = 5.28 \times 10^{27}$ ~~different~~ different groups of 30 are possible.

7. **In problem #6, 8 of the town's 116 people have high mercury levels in their blood. What is the probability that none of these 8 people end up in the sample of 30 selected for study?**

$116 - 8 = 108$ if the 30 are selected from these 108 NO one in the sample will have high mercury in their blood.

$$\frac{_{108}C_{30}}{_{116}C_{30}} = \frac{\text{ways the 30 can come from the 108}}{\text{ways the 30 can come from all 116}}$$

$= 0.0835$