96 points possible

Statistics 300
Instructor: L. C. Larsen

Name: Solution

Summer 2011
Mon/Tue/Wed/Thu 5:30 - 8:15 pm

---

1. (4 points; 5 minutes)
Identify the sampling approach in each situation below as SIMPLE RANDOM, STRATIFIED RANDOM, SYSTEMATIC, CLUSTER, CONVENIENCE, or CENSUS sampling.

(a) **Radio stars earn large salaries. Their earning power, however, depends on the age groups that like to listen to them. Therefore, a marketing company separates the whole population into several age groups and takes a random sample of 1000 people in each age group to study which radio stars are worth the most money.**

stratified random

(b) **A manufacturer of radios wants to start a testing program to improve quality. Over the 52 weeks in each year, radios will be sampled from the production line on Tuesdays and Thursdays of even-numbered weeks at the end of each hour that the production line operates.**

systematic

(c) **People searching for evidence of extra-terrestrial life scan space for non-random patterns of radio waves. People with home computers participate in a massive network to provide the power needed to examine the data. To get a sample of people to participate, the program leaders place ads in science fiction magazines and computer magazines. An 800 number is listed so people can volunteer for the program.**

Convenience

(d) **An auditor must review 15% of 8,955 case files in 180 file drawers. A list of the 8955 cases is prepared and rearranged at random. The first 15% of the cases in the rearranged list are reviewed to see ~~of~~ if they were handled correctly.**

simple random

Statistics 300
Instructor: L. C. Larsen

Name: Solution

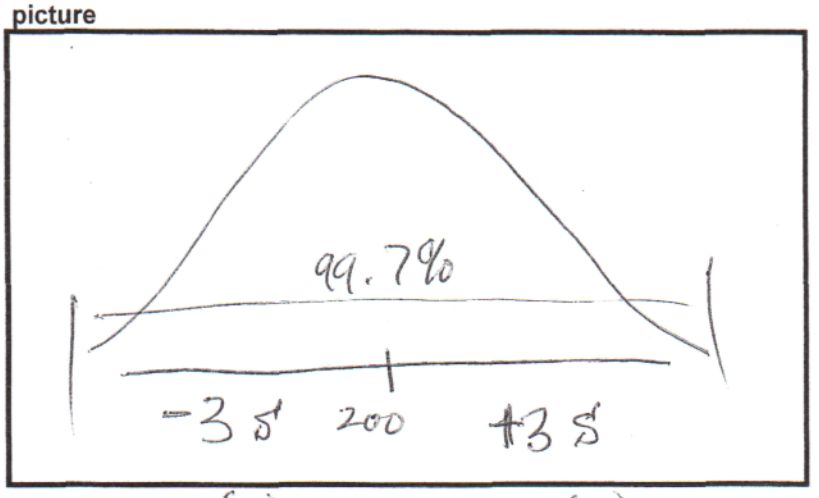
Summer 2011
Mon/Tue/Wed/Thu 5:30 - 8:15 pm

2. (4 points; 5 minutes)
The sample of 170 values below comes from a population with a distribution that is shaped like a bell. First, estimate the standard deviation of the distribution, then estimate the lower and upper values that should include about 99.7% of the values. You may use the "sample statistics" provided in any way you like. Show or describe how you found your answers. Remember, only 4 minutes.

Sample Statistics	
Minimum =	76
Maximum =	316
Mean =	200
Count =	170

Estimates:	
stand. deviation =	60
lower 99.7% limit =	20
upper 99.7% limit =	380

picture



$-3(60)$ $+3(60)$

-180 $+180$

20 380

76	98	111	116	120	124	126	130	131	134
137	139	141	142	144	145	146	148	150	151
152	154	155	156	157	158	159	161	162	163
164	165	166	166	167	168	169	170	170	171
172	173	173	174	175	176	176	177	177	178
179	180	181	181	182	182	183	183	184	185
186	186	187	188	188	189	190	190	191	191
192	193	193	194	194	195	195	196	197	197
198	198	198	199	200	200	201	202	202	203
203	204	205	205	206	207	207	207	208	209
209	210	211	211	212	212	213	214	215	215
216	217	217	218	219	219	220	221	222	222
223	223	224	225	226	226	227	228	229	230
230	231	231	232	233	234	235	235	236	237
238	239	240	241	242	243	244	246	247	248
249	251	252	253	255	256	257	258	260	262
265	268	270	272	275	278	284	289	296	316

estimate st. dev. with range rule: $\frac{\max - \min}{4}$

$= \frac{316 - 76}{4} = 60$

$3 * 60 = 180$

3. (13 points; 15 minutes)
Answer parts a, b, c, and d assuming you know the following things about all of the adults in the United States of America:

32% of all women are college graduates	**===>**	**P(G | W) = 0.32**
17% are both women and college graduates	**===>**	**P(G and W) = 0.17**
12% are both men and college graduates	**===>**	**P(G and M) = 0.12**

(4 points)
(a) What is the probability that a randomly selected adult American will be a woman?

$$P(G|W) = \frac{P(G \text{ and } W)}{P(W)} \qquad P(W) = \frac{0.17}{0.32} = 0.531$$

$$0.32 = \frac{0.17}{P(W)}$$

(2 points)
(b) What is the probability that a randomly selected adult American will be a man?

$$P(M) = 1 - P(W) = 1 - 0.531 = 0.469$$

(4 points)
(c) What is the probability that a randomly selected adult American will be both a man and a college graduate?

$P(G \text{ and } M) = 0.12$ given in problem

(3 points)
(d) Is college graduation in the USA independent of gender, and how did you decide on your answer?

$$P(G|M) = \frac{P(G \text{ and } M)}{P(M)} = \frac{0.12}{0.469} = 0.256$$

But

$P(G|W) = 0.32$

probability of G changes with gender, so "dependent" Not "independent"

Statistics 300
Instructor: L. C. Larsen

Name: Solution

Summer 2011
Mon/Tue/Wed/Thu 5:30 - 8:15 pm

4. Use the 170 values below (in sorted order) **to answer parts (a) and (b).**

(4 points; 3 minutes)

(a) What is the value of the 83rd percentile, P_{83}? $k = 83$; what is the data value?

$$L = \left(\frac{k}{100}\right) N = \left(\frac{83}{100}\right) 170 = 141.1 \rightarrow 142 \text{ location}$$

$$P_{83} = 239$$

(4 points; 3 minutes)

(b) What percentile corresponds to the value 226?

Data value = 226; what is k?

$$k = \left(\frac{\#\text{ of values} < 226}{\text{total \# of values}}\right) 100 = \left(\frac{124}{170}\right) 100 = 72.9$$

$$226 = P_{72.9} \text{ OR } P_{73}$$

76	98	111	116	120	124	126	130	131	134
137	139	141	142	144	145	146	148	150	151
152	154	155	156	157	158	159	161	162	163
164	165	166	166	166	168	169	170	170	171
172	173	173	174	175	176	176	177	177	178
179	180	181	181	182	182	183	183	184	185
186	186	187	188	188	189	190	190	191	191
192	193	193	194	194	195	195	196	197	197
198	198	198	199	200	200	201	202	202	203
203	204	205	205	206	207	207	207	208	209
209	210	211	211	212	212	213	214	215	215
216	217	217	218	219	219	220	221	222	222
223	223	224	225	226	226	227	228	229	230
230	231	231	232	233	234	235	235	236	237
238	239	240	241	242	243	244	246	247	248
249	251	252	253	255	256	257	258	260	262
265	268	270	272	275	278	284	289	296	316

50 100 150

L = 142

Statistics 300
Instructor: L. C. Larsen

Name: Solution

Summer 2011
Mon/Tue/Wed/Thu 5:30 - 8:15 pm

5. (5 points;5 minutes)

A small ferry boat carries people and cars across a river. The boat can carry 10 people and 2 cars. Five cars are waiting to cross the river -- 2 Red (R), 2 Green (G), and one Blue (B). The car owners all claim to have arrived at the same time, so the ferry boat operator decides to pick one car at random and then another car at random to get on the boat.

(a) List the sample space for the cars using the boat operators' procedure (e.g. $\{G_2,G_1\}$)

R_1R_2	R_2R_1	G_1R_1	G_2R_1	BR_1
R_1G_1	R_2G_1	G_1R_2	G_2R_2	BR_2
R_1G_2	R_2G_2	G_1G_2	G_2G_1	BG_1
R_1B	R_2B	G_1B	G_2B	BG_2

(b) Because the ferry operator picks at random, what is the probability he will pick at least one of the Red cars?

The circled events have at least one red car.

14 circled events out of 20.

$$P(\text{at least one red}) = \frac{14}{20}$$

Statistics 300
Instructor: L. C. Larsen

Name: Solution

Spring 2011
Tue/Thu 7:00 - 9:05 pm

6. (6 points; 6minutes)
Given:
X is distributed Binomial where n = 2000 and p = 0.72
Y is distributed Binomial where n = 800, p = 0.44

Which would be more unusual, X = 1392 or Y = 378 ?

$\mu_X = (2000)(0.72) = 1440$

$\sigma_X = \sqrt{2000(.72)(.28)} = 20.08$

$Z_X = \dfrac{1392 - 1440}{20.08} = -2.39$

$\mu_y = 800(0.44) = 352$

$\sigma_y = \sqrt{800(0.44)(0.56)} = 14.04$

$Z_Y = \dfrac{378 - 352}{14.04} = 1.85$

$|Z_X| > |Z_Y|$, so $X = 1392$ is more unusual

7. (7 points; 7 minutes)
Complete the columns in the "Frequency Distribution" table using the data values given below.

Frequency Distribution

Class Limits						
Lower	Upper	Tally	Frequency	Relative Frequency	Cumulative Frequency	Cumulative Relative Frequency
0.00	0.04	𝍸	5	5/11	5	5/11
0.05	0.09	𝍸	3	3/11	8	8/11
0.10	0.14	𝍸	3	3/11	11	11/11

Class Boundary	Class Midpoint
	0.02
0.045	0.07
0.095	0.12

$n = 11$

Data:
~~0.073~~ ~~0.038~~ ~~0.043~~ ~~0.032~~ ~~0.128~~ ~~0.006~~
~~0.066~~ ~~0.014~~ ~~0.138~~ ~~0.054~~ ~~0.097~~

Statistics 300
Instructor: L. C. Larsen

Name: Solution

Spring 2011
Tue/Thu 7:00 - 9:05 pm

8. (8 points; 7 minutes)
For the two situations below, determine whether each is a proper (valid) discrete probability distribution. If it is a proper distribution, determine the mean, variance, and standard deviation.

(a)

x	P(x)	$x \cdot P(x)$	$(x-\mu)^2 \cdot P(x)$	
17	0.66	11.22	179.9	
60	0.25	15.00	175.4	
81	0.09	7.29	203.0	
$\Sigma = 1$ "valid"		$\mu = 33.51$	558.3 $= \sigma^2$	$\sqrt{558.3} = 23.63 = \sigma$

μ	σ^2	σ
formula $= \Sigma x \cdot P(x)$	formula $= \Sigma (x-\mu)^2 \cdot P(x)$	formula $= \sqrt{\Sigma (x-\mu)^2 \cdot P(x)}$
value	value	value

(b)

x	P(x)			
17	0.66			
60	0.15			
81	0.09			
$\Sigma = 0.9$ "Not Valid"				

μ	σ^2	σ
formula	formula	formula
value	value	value

Statistics 300
Instructor: L. C. Larsen

Name: Solution

Spring 2011
Tue/Thu 7:00 - 9:05 pm

9. (14 points; 14 minutes)
For the data in this problem, provide a definition for each sample statistic, then calculate the value of each statistic for sample of data listed in the problem. Use your calculator's statistical functions where possible; "by hand" is not acceptable for the mean and the standard deviation.

Data:

~~70~~
~~62~~
~~64~~
~~63~~
~~66~~
~~65~~
~~62~~
~~67~~

62
62
63
64
65
66
67
70

+÷2 = 64.5

	Definition or Formula	Value for these data
RANGE	Max − Min	70 − 62 = 8
MODE	Most frequently occurring value	62
STANDARD DEVIATION	$\sqrt{\frac{\Sigma(x-\bar{x})^2}{n-1}} = s$	$s = 2.748$
MEAN	$\frac{\Sigma x}{n}$	$\bar{x} = 64.875$
VARIANCE	$\frac{\Sigma(x-\bar{x})^2}{n-1} = s^2$	$s^2 = 7.552$
MEDIAN	Value in the middle when data are in sorted order	(64+65)/2 = 64.5
MIDRANGE	$\frac{\text{Min} + \text{Max}}{2}$	(62 + 70)/2 = 66

Statistics 300
Instructor: L. C. Larsen

Name: Solution

Spring 2011
Tue/Thu 7:00 - 9:05 pm

10. (4 points: 5 minutes)

A business analyst determines that a new store will earn $100,000 in profits if it is the only store in the area. But, it will only earn $60,000 if another store opens, and it will lose $20,000 of two more stores open. The probability that the new store will be the only store is 55%. The probability that only one other store will open is 35%. And, the probability that two other stores will open is 10%.

Expected Value = $\Sigma x \cdot P(x)$

What is the <u>expected value</u> of a decision to open the store?

earnings

x	$P(x)$	$x \cdot P(x)$
-20000	0.10	-2000
60000	0.35	21000
100000	0.55	55000
		Σ = $74,000 = expected value

11. (8 points; 5 minutes)

Circle the correct choice in each box based on <u>the underlined text</u> in each part.

a. **<u>A dog, a cat, and a bird</u> compete in three races – a short one, a medium one, and a long one. The amount of energy used by each animal in each race is measured. The bird won 2 races, the dog 1, and the cat 0.**

Are the data ... ?	Are the data ... ?	
(Qualitative)	(Nominal)	Interval
Quantitative and Discrete		
Quantitative and continuous	Ordinal	Ratio

b. **A dog, a cat, and a bird compete in three races – <u>a short one, a medium one, and a long one</u>. The amount of energy used by each animal in each race is measured. The bird won 2 races, the dog 1, and the cat 0.**

(Qualitative)	Nominal	Interval
Quantitative and Discrete		
Quantitative and continuous	(Ordinal)	Ratio

c. **A dog, a cat, and a bird compete in three races – a short one, a medium one, and a long one. <u>The amount of energy used</u> by each animal in each race is measured. The bird won 2 races, the dog 1, and the cat 0.**

Qualitative	Nominal	Interval
Quantitative and Discrete		
(Quantitative and continuous)	Ordinal	(Ratio)

d. **A dog, a cat, and a bird compete in three races – a short one, a medium one, and a long one. The amount of energy used by each animal in each race is measured. <u>The bird won 2 races, the dog 1, and the cat 0</u>.**

Qualitative	Nominal	Interval
(Quantitative and Discrete)		
Quantitative and continuous	Ordinal	(Ratio)

12. (3 points : 3 minutes)
A store sells five different types of home appliances – refrigerators (R), washing machines (W), dryers (D), stoves (S), and automatic dishwashers (A). How any different ways could the next 10 sales happen? (Example: W,S,R,R,D,W,D,W,R,A) 10 sales

$5 \quad 5 \quad 5 \quad \cdots \quad 5 = 5^{10} =$ (9,765,625)

13. (3 points : 3 minutes)
A store sells five different types of home appliances – refrigerators (R), washing machines (W), dryers (D), stoves (S), and automatic dishwashers (A). How many different ways could the sales manager arrange one of each type of appliance along the back wall to show customers?

RWDSA all arrangements $= 5!$ or ${}_5P_5$

or $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 =$ (120) ways

14. (3 points : 3 minutes)
A store that sells five different types of home appliances – refrigerators (R), washing machines (W), dryers (D), stoves (S), and automatic dishwashers (A) – employs 20 people on the sales floor. Management has decided to select 5 of the 20 to take part in special training. How many different groups of 5 could management choose?

order does not count

${}_{20}C_5 =$ (15,504)

15. (6 points; 5 minutes)
A company purchases thousands of items every day for their business. Tomorrow, 82% of the items they will receive will be "good" and the other 18% will be "bad". If the company selects 8 items at random tomorrow, what is the probability that exactly 5 of the 8 will be "good"?

- Number of trials = 8 // "8 out of thousands" (independent)
- 2 possible results of each trial "good" and "bad"
- prob(good) $= p$ for all trials $= 0.82$ Prob(bad) $= 1 - p = q = 0.18$

$P(\text{exactly 5 good}) = {}_nC_x (p)^x (q)^{n-x}$

$= {}_8C_5 (0.82)^5 (0.18)^3 =$ (0.121)