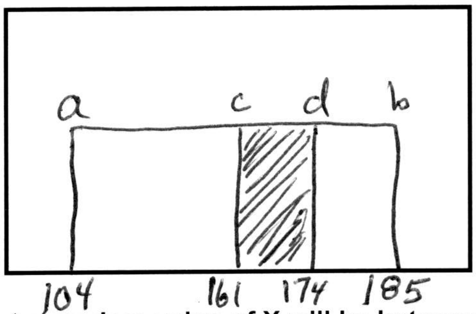
(4 points)

1. A random variable, X, has a Uniform distribution on the interval [104,185].

   (a). Draw the distribution in this box.



   (b). What is the probability that a random value of X will be between 161 and 174?

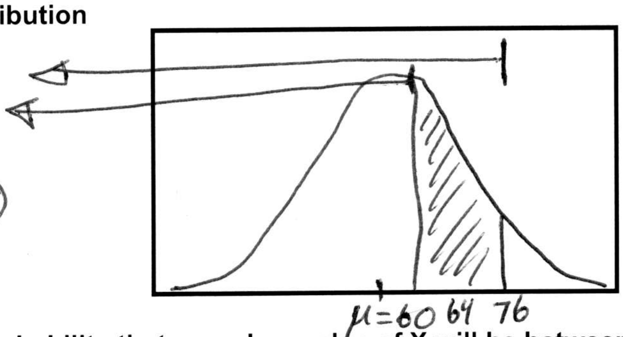
$$P(161 < X < 174) = \frac{d-c}{b-a} = \frac{174-161}{185-104} = \frac{13}{81} = 0.160$$

(5 points)

2. Given: X has a Normal distribution with μ = 60 and σ = 12.

   (a). Draw the distribution in this box.

$$\begin{array}{r} .9082 \\ -.6293 \\ \hline 0.2789 \end{array}$$



$$Z_1 = \frac{76-60}{12} = \frac{16}{12} = 1.33$$

$$Z_2 = \frac{64-60}{12} = \frac{4}{12} = 0.33$$

   (b). What is the probability that a random value of X will be between 64 and 76?

$\sigma = 12$

$$P(64 < X < 76) = 0.2789$$

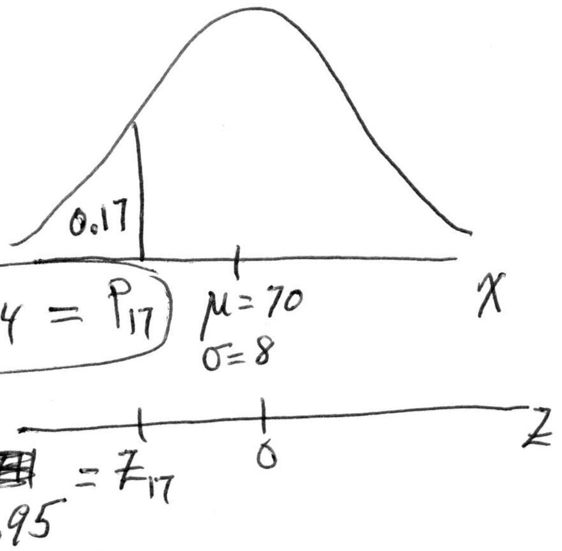
(5 points)

3. Given: X ~ N(μ = 70, σ = 8).
   What is the 17th percentile of this distribution?
   (draw the picture for safety's sake)

$$\frac{P_{17} - 70}{8} = Z_{17} = -0.95$$

$$P_{17} = (-0.95)8 + 70$$

$$= 62.4$$

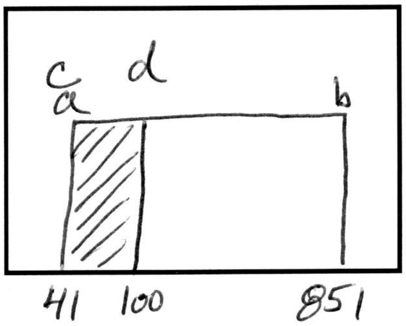


1

(4 points: 3 minutes)

4. Given: X~U[41,851].

(a). Draw the distribution of X in this box.



(b). What is the probability that a random value of X will be less than 100?

$$P(X<100) = \frac{d-c}{b-a} = \frac{100-41}{851-41}$$

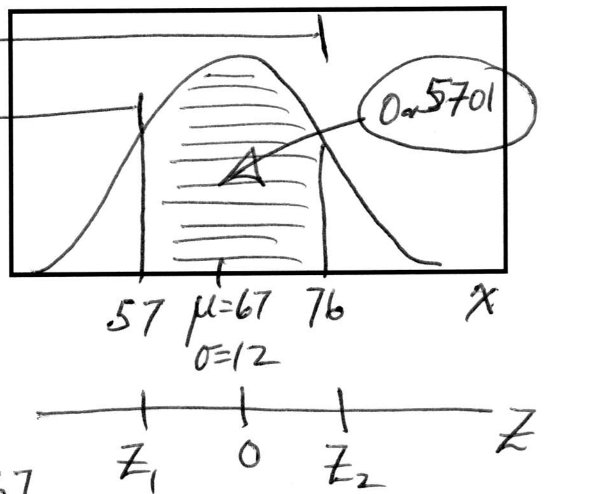
$$= \frac{59}{810} = 0.0728$$

(5 points: 4 minutes)

5. Given: X has a Normal distribution with μ = 67 and σ = 12.

(a). Draw the distribution in this box.

$$\begin{array}{r} 0.7734 \\ -\ 0.2033 \\ \hline 0.5701 \end{array}$$



(b). What is the probability that a random value of X will be between 57 and 76?

	.03
−0.8	.2033

	.05
0.7	.7734

$$Z_1 = \frac{57-67}{12} = \frac{-10}{12} = -0.83$$

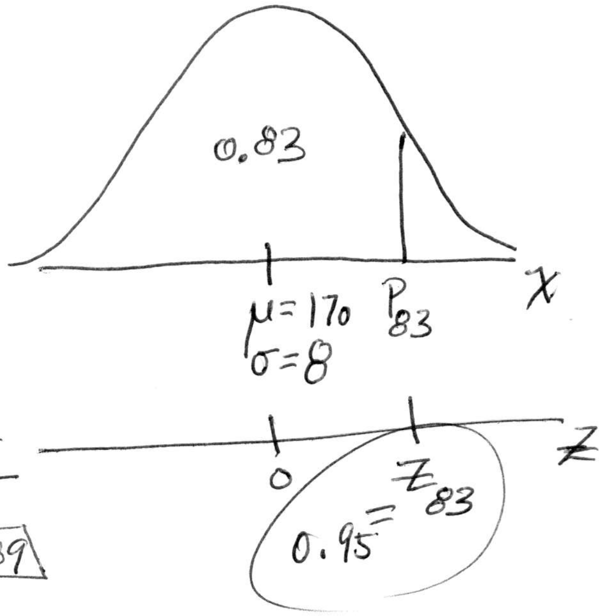
$$Z_2 = \frac{76-67}{12} = 0.75$$

(5 points: 4 minutes)

6. Given: X~N(μ = 170 and σ = 8).

What is the 83rd percentile of this distribution?

(draw the picture for safety's sake)



$$\frac{P_{83}-\mu}{\sigma} = Z_{83}$$

$$\frac{P_{83}-170}{8} = 0.95$$

$$P_{83} = (0.95)8 + 170 = 177.6$$

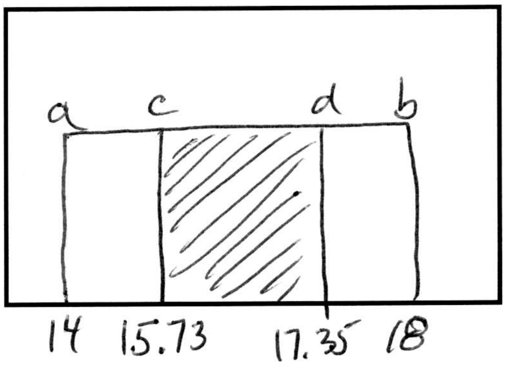
	.05
0.9	.8289

2

(4 points)

5. **A random variable, X, has a Uniform distribution on the interval [14,18]. What is the probability that a random value from this distribution will be between 15.73 and 17.35?**

   Draw the picture for the problem and calculate the probability.



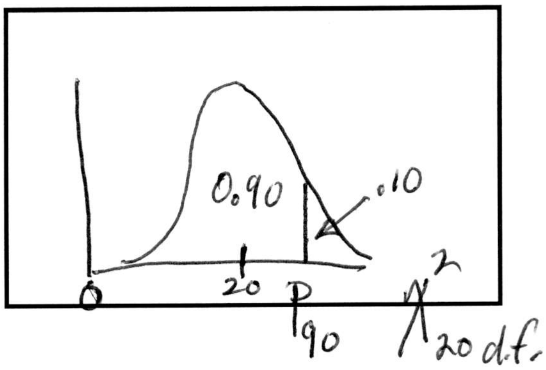
$$P(15.73 < X < 17.35) = \frac{17.35 - 15.73}{18 - 14}$$

$$= \frac{1.62}{4} = 0.405$$

(3 points : 3 minutes)

6. **A random variable, X, has a Chisquare distribution with 20 degrees of freedom. What is the 90th percentile of this distribution?**

   Draw the picture for the problem and determine the value of the percentile.

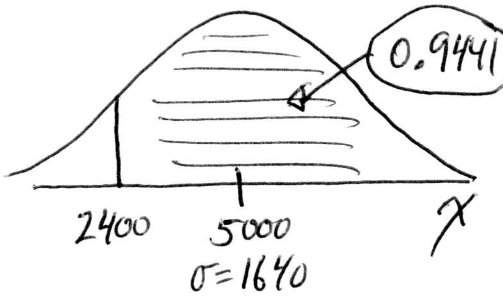


df.	Area to Right
	0.10
20	$\boxed{28.412}$

$$P_{90} = 28.412$$

Pumpkins have weights that are normally distributed with an average of 5000 grams and a standard deviation of 1640 grams.

(a) If stores will only buy pumpkins that weigh more than 2400 grams, what proportion (or percent) of pumpkins will the stores buy?



$$z = \frac{2400 - 5000}{1640} = \frac{-2600}{1640} = -1.59$$

Area to right of -1.59

= Area to left of $+1.59$ = 0.9441

	.09
1.5	$\boxed{0.9441}$

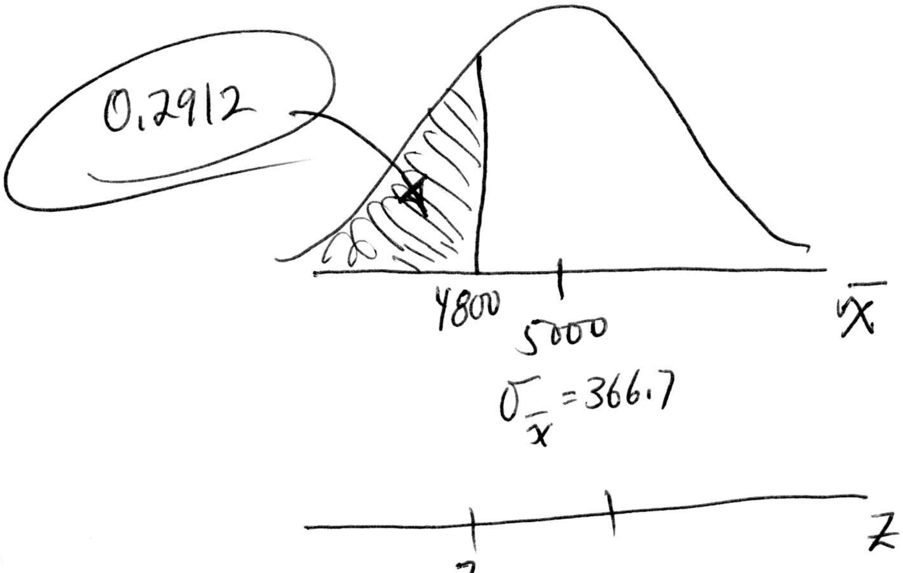
(b) If a store buys 20 pumpkins that are randomly selected, what is the probability that the average weight of the 20 pumpkins will be less than 4800 grams?

for all possible samples of size 20, the distribution of the sample means is approximately

$$\bar{x} \sim N\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$$

$$\sim N\left(5000, \frac{1640}{\sqrt{20}}\right)$$

$$\sim N(5000, 366.7)$$



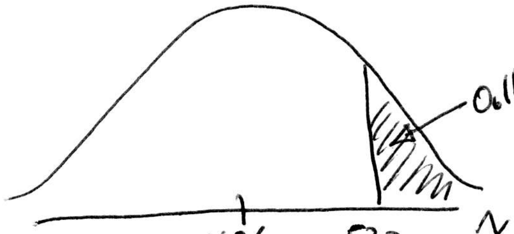
	.05
−0.5	$\boxed{.2912}$

$$z = \frac{4800 - 5000}{366.7} = -0.55$$

(10 points; 9 minutes)

4. **Packages of carrots have weights that are normally distributed with an average of 486 grams and a standard deviation of 45 grams.**

(a) What percent of the packages have weights <u>greater than</u> 530 grams?



0.1635

$P(x > 530) = 0.1635$

486 530 x

$\sigma = 45$

$0.98 = z = \dfrac{530 - 486}{45}$

(b) If 10 packages of carrots are randomly selected, what is the probability that their average weight will be less than 500 grams?

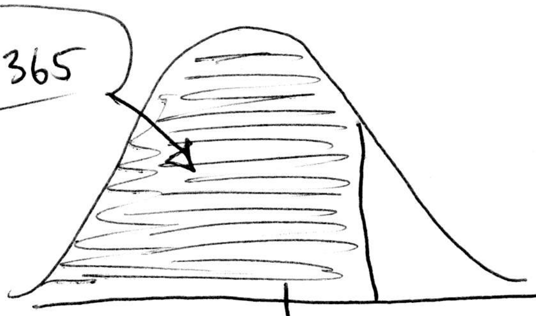
$x \sim N(486, 45)$

for $n = 10$

$\bar{x} \sim N\left(486, \dfrac{45}{\sqrt{10}}\right)$

$\sim N(486, 14.23)$

0.8365



486 500 $\bar{x}$

$\sigma_{\bar{x}} = 14.23$

0 z

$\dfrac{500 - 486}{14.23} = 0.98$

	.08
0.9	$\boxed{.8365}$

(6 points)

5. **A zoologist studies the gestation period (days to birth) for a newly discovered variety of porcupine. A random sample of 8 of the purcupines provides gestation periods with an average of 96 days and a standard deviation of 6 days. Gestation periods usually have a distribution that is sort of bell-shaped.**

   Use the data for this sample of porcupines to construct a 95% confidence interval for the true mean gestation period for the new variety of porcupines.

$n = 8$

$\bar{x} = 96$

$s = 6$

$d = 7$

confidence $= 0.95$

$\alpha = 0.05$ in 2 tails

$t_{\alpha/2} = 2.365$

$$95\%\ CI(\mu) = \bar{x} \pm t_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right)$$

$$= 96 \pm 2.365\left(\frac{6}{\sqrt{8}}\right)$$

$$= 96 \pm (2.365)(2.12)$$

$$= 96 \pm 5.0$$

$$= [91 < \mu < 101]$$

The local water treatment plant tests samples of water to see if the average water quality meets established standards. The plant operators use a new test for mercury which is more sensitive than their usual test. Eighteen (18) samples are taken at random and measured for mercury. Use the test results given in the box to construct a 90% confidence inteval estimate for the mean mercury content of all the water the plant treats.

n =	18
$\bar{x}$ =	28 ppb
s =	5.9 ppb

df = 17

confidence = 0.90

$\alpha = 0.10$ in 2 tails

$t_{\alpha/2} = 1.740$

$$90\%\ CI(\mu) = \bar{x} \pm t_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right)$$

$$= 28 \pm 1.740\left(\frac{5.9}{\sqrt{18}}\right)$$

$$= 28 \pm (1.740)(1.39)$$

$$= 28 \pm 2.4$$

$$= [25.6 < \mu < 30.4]$$

(6 points)

8. A random sample of 220 voters on election day shows your candidate with 47 percent of these votes. Use these results to construct a 95% confidence interval for the percent of the vote your candidate will get when all of the votes are counted.

$n = 220$

$\hat{p} = 0.47$

$\hat{q} = 0.53$

Confidence $= 0.95$

$\alpha = 0.05$

$\alpha/2 = 0.025$

$Z_{\alpha/2} = 1.96$

$$95\%\,CI(p) = \hat{p} \pm Z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$= 0.47 \pm 1.96\sqrt{\frac{(0.47)(0.53)}{220}}$$

$$= [0.404 < p < 0.536]$$

(6 points: 6 minutes)

1. A random sample voters before an election shows your candidate with 144 votes and the opposing candidate with 81 votes. Use these results to construct a 98% confidence interval for the percent of the vote your candidate will get when all of the votes are counted.

$$\begin{array}{r} 144 \\ 81 \\ \hline 225 \end{array} = n$$

$\frac{144}{225} = \hat{p} = 0.64$

$\hat{q} = 0.36$

confidence $= 0.98$

$\alpha = 0.02$

$\alpha/2 = 0.01$

$Z_{\alpha/2} = 2.33$

$$98\%\,CI(p) = \hat{p} \pm Z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$= 0.64 \pm 2.33\sqrt{\frac{(0.64)(0.36)}{225}}$$

$$= 0.64 \pm 2.33(0.032)$$

$$= 0.64 \pm 0.075$$

$$= [.565 < \hat{p}\ 0.715]$$

A pharmaceutical company wants to know the percentage of people that use their product. A survey of 884 randomly selected people found that 13% had used the product in the last year. Use this information to make a 95% confidence interval for the percentage of all people that use the company's product.

$n = 884$

$\hat{p} = 0.13$

$\hat{q} = 0.87$

confidence $= 0.95$

$\alpha/2 = 0.025$

$Z_{\alpha/2} = 1.96$

$$95\%\, CI(p) = \hat{p} \pm Z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$= 0.13 \pm 1.96\sqrt{\frac{(0.13)(0.87)}{884}}$$

$$= 0.13 \pm 0.022$$

$$= [0.108 < p < 0.152]$$

Based on your confidence interval, is it reasonable for the company to put an advertisement on TV saying that 1 out of 5 people use their product? Why did you chose your answer?

Circle : Yes No

Why? 1 out of 5 $= \frac{1}{5} = 0.2$. This value is not in the CI, which is the reasonable range.

(8 points : 8 minutes)

2. In a random sample of 647 adult men who had been in Boy Scouts for more than one year, 62 had achieved the rank of "Eagle Scout" and the rest had not achieved that rank. Use these data to make a 95% confidence interval for the true proportion of all adult men who had been in Boy Scouts for more than one year and had achieved the rank of Eagle Scout.

$n = 647$

$x = 62$

$\hat{p} = \frac{x}{n} = \frac{62}{647} = 0.0958$

$\hat{q} = 1 - \hat{p} = 0.9042$

$Z_{\alpha/2} = 1.96$

$95\%\ CI(p) = \hat{p} \pm Z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$

$= 0.0958 \pm 1.96\sqrt{\frac{(.0958)(.9042)}{647}}$

$= 0.0958 \pm 0.0116$

$= [0.084 < p < 0.107]$

Based on your results, is it reasonable for someone to claim that only 10% or less of all men who spent more than one year in Boy Scouts were able to earn the rank of Egle Scout? Circle "Yes" or "No" and say why.

YES **NO** Because values that are "10% or less" are included in the CI, which is the reasonable range.

10

A quality control supervisor tracks the variation in the thickness of ciruit boards used to make computers. A systemmatic sample (treat it as if it was random) of 41 circuit boards made last week were measured for thickness and the results are shown in the box below. Use the results to construct a 98% confidence interval for the standard deviation of the thicknesses of all 48,000 of the circuit boards produced last week.

$n =$	41
$\bar{x} =$	2.875 mm
$s =$	0.053 mm

$df = 40$

$\alpha = 0.02$

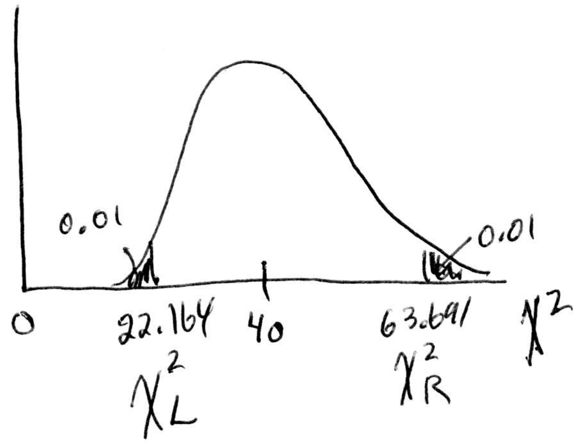
$\chi^2_L = 22.164$

$\chi^2_R = 63.691$

98% CI(σ): $\sqrt{\dfrac{(n-1)s^2}{\chi^2_R}} < \sigma < \sqrt{\dfrac{(n-1)s^2}{\chi^2_L}}$

$$\sqrt{\frac{(41-1)(0.053)^2}{63.691}} < \sigma < \sqrt{\frac{(41-1)(0.053)^2}{22.164}}$$

$$[0.042 < \sigma < 0.071]$$



(8 points)

9. **A company supplies an engine part to a major automobile manufacturing client. All of the parts must be very uniform in length to be acceptable. A random sample of 19 parts is selected and the length of each one is measured. The results are summarized below. Use these results to prepare a 95% confidence interval for the standard deviation of the lengths of all the parts this company manufactures.**

Results from sample	
10.376 cm	**= average**
0.0021 cm	**= stand. deviation**
19	**= sample size**

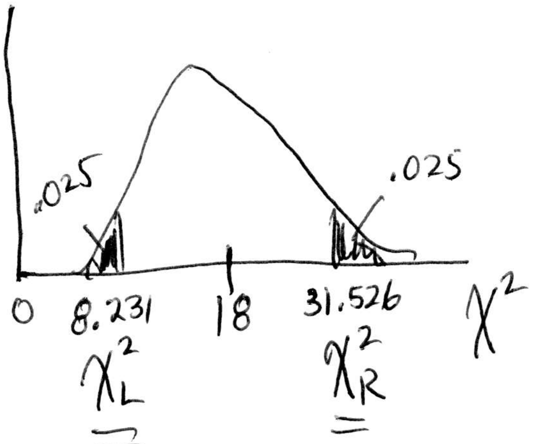
95% CI(σ): $\sqrt{\dfrac{(n-1)s^2}{\chi^2_R}} < \sigma < \sqrt{\dfrac{(n-1)s^2}{\chi^2_L}}$

df = 18

confidence = 0.95

$\alpha/2 = 0.025$ in each tail

$$\sqrt{\frac{(19-1)(.0021)^2}{31.526}} < \sigma < \sqrt{\frac{(19-1)(.0021)^2}{8.231}}$$



[0.0016 < σ < 0.0031]

(7 points: 7 minutes)

11. A fast food restaurant wants the coffee it serves to be hot enough, but not too hot. The management's goal is for the coffee temperatures to average 150 °F with a standard deviation less than 4 °F for all coffee served. The management randomly samples 24 cups of coffee and measures the temperature of each. The average of the 24 cups is 148.7 °F and the standard deviation is 6.2 °F. Use this information to construct a 95% confidence interval for the true standard deviation (σ) of temperatures for all coffee served at this restaurant.

$n=24$

$df=23$

$s=6.2$

$\chi^2_L = 11.689$

$\chi^2_R = 38.076$

95% CI(σ): $\sqrt{\dfrac{(n-1)s^2}{\chi^2_R}} < \sigma < \sqrt{\dfrac{(n-1)s^2}{\chi^2_L}}$

$$\sqrt{\frac{(24-1)(6.2)^2}{38.076}} < \sigma < \sqrt{\frac{(24-1)(6.2)^2}{11.689}}$$

$$[4.82 < \sigma < 8.70]$$

Based on your confidence interval, do you think the management's goal of a standard deviation less than 4 °F is being achieved?

(yes or no, with reason that justifies your answer)

No. Because all of the values in the CI(σ) are greater than 4°F. Values from 4.82 through 8.70 are the reasonable range for σ.

(6 points)

1. Give the appropriate "null hypothesis" and "alternative hypothesis" for the following cases: Show each hypothesis symbolically (e.g., H_0: $\sigma < 200$).

(a) A medical auditor for a major hospital in the eastern United States believes his company is undercharging the Medicare system. The average of all Medicare charges is $1850 per patient. Set up the hypotheses to test.

Nationally

H_0: $\mu \geq 1850$

claim or belief: $\mu < 1850$

for this company.

H_1: $\mu < 1850$

(b) A medical investigator claims that a major drug producer in the central United States has poor quality control, causing the potency of one "generic" drug to have excessive variability. The maximum standard deviation allowed is 0.25 milligrams (mg).

H_0: $\sigma \leq 0.25$

claim: $\sigma > 0.25$

H_1: $\sigma > 0.25$

(c) A medical regulator is responsible for certifying drugs as safe. He or she must decide how the percentage of people who have serious side effects when using a new drug compares with the percentage that has serious side effects when using the current standard drug. The percentage for the current standard drug is 1.8%.

H_0: $p = 0.018$

compares to: $<, =, >$

H_1: $p \neq 0.018$

(6 points)

2. **For the following situations, draw the critical region and show the critical values:**

(a) $H_0: \mu <= 1000$ ≤

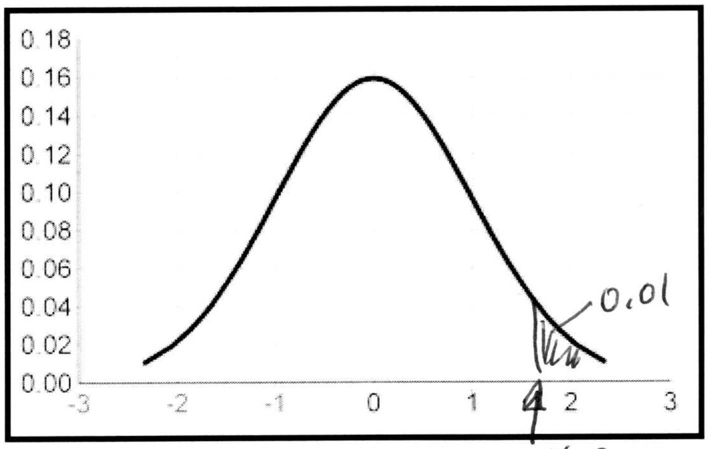
$H_1: \mu > 1000$ right tail

$\alpha = 0.01$

$n = 28$

df= 27

t = 2.473



2.473

(b) $H_0: \mu = 1.84$

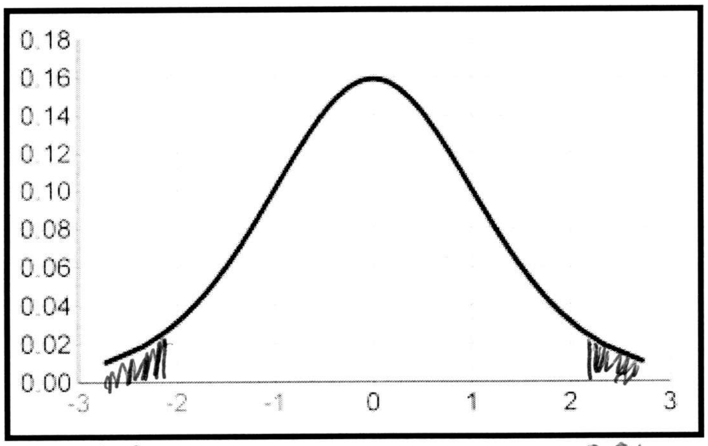
$H_1: \mu <> 1.84$ ≠

$\alpha = 0.05$ 2 tails

$n = 12$

df=11

t =2.201



−2.201 2.201

(c) $H_0: \mu => 19.2$ ≥

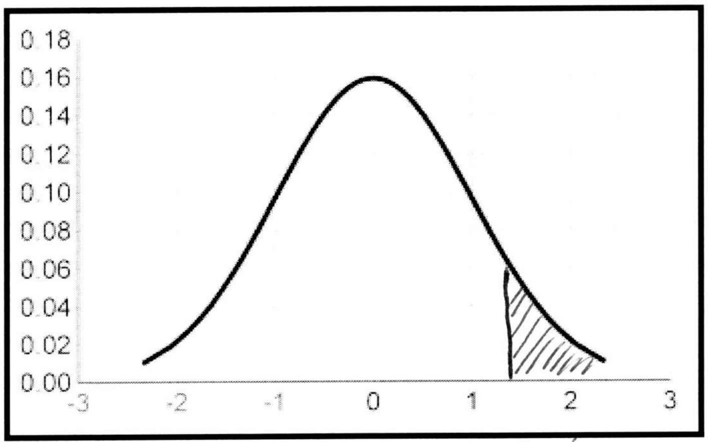
$H_1: \mu < 19.2$ left tail

$\alpha = 0.10$

$n = 61$

df=60

t=1.296



1.296

(8 points)

9. **A random sample of 16 Zoologists discovers that their average weight is 100 kg with a standard deviation of 22 kg. Use the sample data to test the proposition that Zoologists weigh less on average that Physicists, whose average weight is known to be 103 kg. (Use α = 0.05)**

claim: $\mu < 103$

sample data
n = 16
$\bar{x}$ = 100 kg
s = 22 kg

H_0: $\mu \geq 103$ OR $\mu = 103$

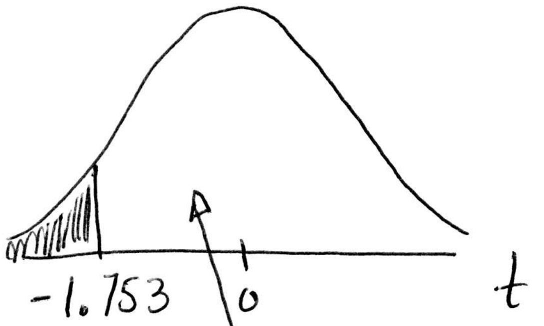
H_1: $\mu < 103$

$\alpha = 0.05$ left tail

df = 15

$\boxed{t = 1.753}$ from table

critical region



test statistic

$$\frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

$$= \frac{100 - 103}{22/\sqrt{16}}$$

$$= \frac{-3}{5.5}$$

$$= -0.55$$

conclusion:

Do not reject H_0.

(9 points: 10 minutes)

2. A sample of 6 felony convictions is selcted at random and the length of the sentence is recorded. Use the data shown for these convictions to test the claim that the lengths of all felony convictions in California average less than 5 years. (Use $\alpha = 0.05$)
(Earlier studies have shown the distribution of sentence lengths is bell-shaped.)

claim: $\mu < 5$

Convict (Subject)	Sentence (years)
1	5
2	6
3	4
4	2
5	3
6	7

H_0: $\mu = 5$ OR $\mu \geq 5$

H_1: $\mu < 5$

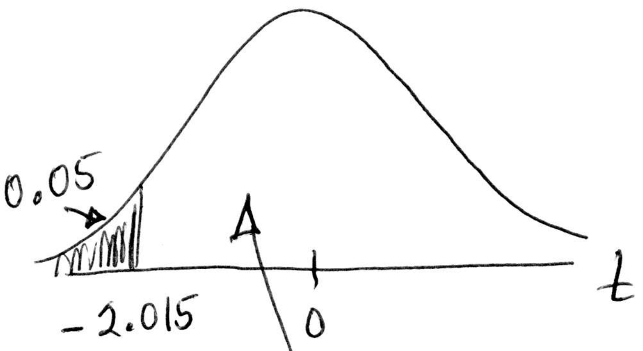
$\alpha = 0.05$ left tail

(Use calculators appropriately.)

$n = 6$ $\bar{x} = 4.5$ $s = 1.871$

$df = 5$ $t = 2.015$

critical region



test statistic

$$\frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

$$= \frac{4.5 - 5}{1.871/\sqrt{6}}$$

$$= \frac{-0.5}{0.76}$$

$$= -0.66$$

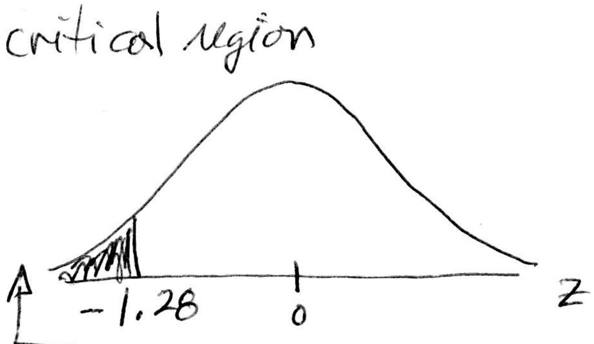
conclusion:

Do not reject H_0.

$\hat{p} = \frac{860}{1000} = 0.86$

A company produces vegetable seeds for gardeners and they advertise that at least 90% of the seeds are viable, so they will sprout when planted and watered. When 1000 randomly selected seeds are tested, 860 of the seeds sprout. Use these results to test the company's advertized claim.
(Use a 10% significance level for your test.)

Q



Claim : $p \geq 0.90$

H$_0$: $p \geq 0.90$

H$_1$: $p < 0.90$

$\alpha = 0.10$ left tail

Test Statistic

$$\frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{0.86 - 0.90}{\sqrt{\frac{(.90)(.10)}{1000}}} = \frac{-0.04}{0.0095} = \boxed{-4.21}$$

$\boxed{\text{Reject } H_0}$

What is the probability of a "Type I" error for this test? **P(Type I error) =** 0.10

$= \alpha$

The sales manager for M&M candies claims that "peanut" M&M's are less popular today than they were ten years ago. In 1996, 84% of all people "loved" peanut M&M's. A survey of 880 people in 2006 found that 78% "love" peanut M&M's. Use these results to test the sales manager's claim. (Use a 0.02 significance level for this test)

claim: $p < 0.84$

Now (2006)

10 years ago (1996)

$n = 880$

$\hat{p} = 0.78$

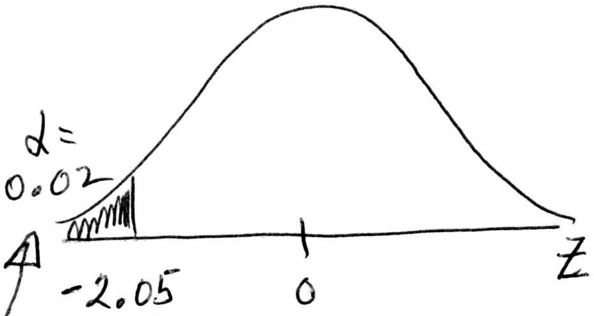
$\alpha = 0.02$

$H_0: p \geq 0.84$

$H_1: p < 0.84$

$\boxed{\alpha = 0.02 \text{ left tail}}$

Critical Region



z	.05
-2.0	$\boxed{.0202}$

Test Statistic

$$\frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}}$$

$(p_0 = 0.84)$

$(q_0 = 0.16)$

$$= \frac{0.78 - 0.84}{\sqrt{\frac{(.84)(.16)}{880}}}$$

$$= \frac{-0.06}{0.0124}$$

$$= -4.84$$

conclusion:

Reject H_0

(8 points)

12. A company produces yummy popcorn for movie theaters. The company advertises that more than 98% of their popcorn kernels will pop when cooked according to instructions. When a random sample of 10,000 kernels was tested under standard conditions by Consumer Reports Magazine, 187 did not pop and 9813 did pop. Use these results to test the company's advertized claim.

(Use α = 10% for your test.)

$n = 10{,}000 \quad x = 9813$

$\hat{p} = 0.9813$

$p_0 = 0.98 \quad q_0 = 0.02$

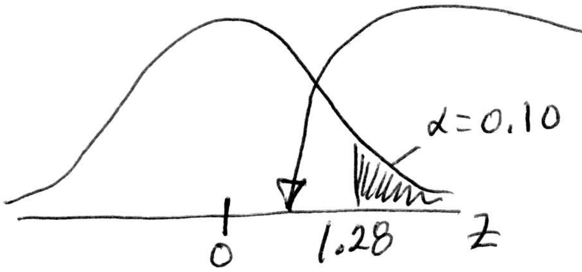
Claim : $p > 0.98$

H0: $p \leq 0.98$ OR $p = 0.98$

H1: $p > 0.98$

$\alpha = 0.10$ in right tail

critical region



Test statistic

$$\frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{0.9813 - 0.98}{\sqrt{\frac{(.98)(.02)}{10000}}} = 0.93$$

What is the probability of a "Type I" error for this test? **P(Type I error) =** $0.10 = \alpha$

conclusion: Do not reject Ho;

(8 points : 10 minutes)

1. **The usual standard deviation of daily power usage for a business is 20 kilowatt hours. For the last thirty days, the standard deviation of daily power usage has been 26 kilowatt hours. Test the claim that the variability in power usage during the last thirty days represents a random sample of days from the usual distribution of daily power usage; that is, the variation is the same as it always has been. Experience says that the usual distribution follows a bell-shaped curve.**
   For this test, use a 0.05 significance level.

H_0: $\sigma = 20$

H_1: $\sigma \neq 20$

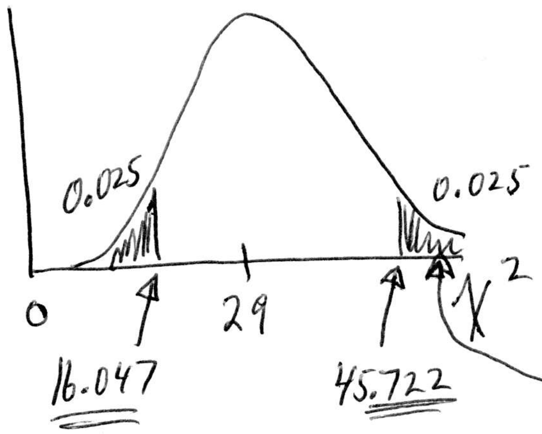
$\alpha = 0.05$ 2 tails

claim: $\sigma = 20$ (the usual)

$n = 30$ (the last 30 days)

$s = 26$ df = ~~[illegible]~~ 29

critical region



Test Statistic

$$\frac{(n-1)s^2}{\sigma_0^2} = \frac{(29)(26)^2}{20^2}$$

$= 49.01$

conclusion:

Reject H_0:

(8 points)

12. For the last 20 years, the std. deviation (σ) of the profits at gasoline stations in California has been \$2.00 per 100 gallons. This year, a sample of 24 gasoline stations gives a standard deviation of \$0.82. Test the claim that the standard deviation of all profits is smaller this year than in the last 20 years (a possible indication of illegal price fixing by gasoline sellers). (Let α = 0.01 for this test.)

claim: $\sigma < 2.00$

Now — 20-year norm

H_0: $\sigma \geq 2.00$

H_1: $\sigma < 2.00$

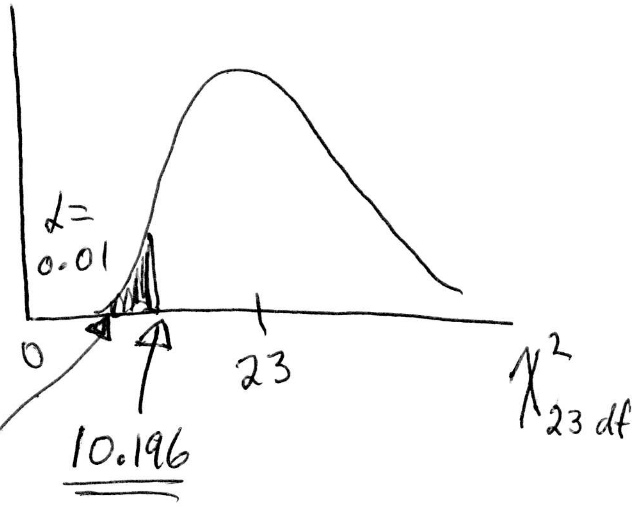
$\alpha = 0.01$ left tail

$n = 24$

$s = 0.82$

$df = 23$

critical region



test Statistic

$$\frac{(n-1)s^2}{\sigma_0^2} = \frac{(24-1)(0.82)^2}{(2.00)^2}$$

$= 3.866$

conclusion:

Reject H_0:

(8 points: 9 minutes)

small σ desired

7. **A mill operator buys wheat to grind into flour for bread. The moisture (water content) of the wheat affects how it grinds. The operator wants wheat with a uniform water content, so the equipment will not need to be readjusted very often. In past years, the standard deviation of the water content has been 1.38. This year, the first 20 batches of wheat purchased have water contents with a standard deviation of 2.44. Assuming these 20 batches of wheat are a random sample from the population of all possible batches for this year, test the claim that the variability of water content is greater than in previous years.** (Use a 10% significance level for this test.)

Now   previous Norm

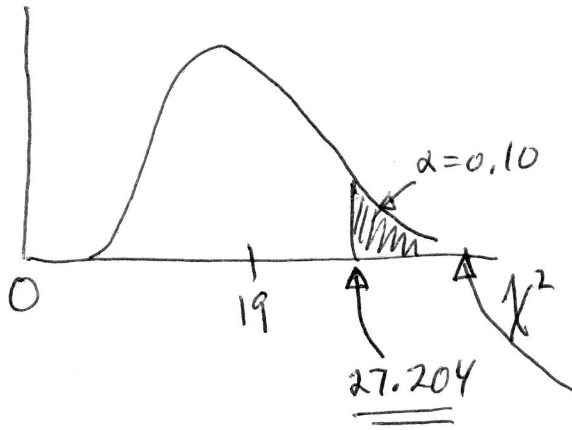
Claim : $\sigma > 1.38$ $n = 20$

H0: $\sigma \le 1.38$ $s = 2.44$

H1: $\sigma > 1.38$ $df = 19$

$\alpha = 0.10$ right tail

critical region



test statistic

$$\frac{(n-1)s^2}{\sigma_0^2} = \frac{(20-1)(2.44)^2}{(1.38)^2}$$

$$= 59.398$$

Conclusion:

Reject Ho:

You are designing an experiment to determine how long in takes for concrete to dry. You decide to use a random sampling procedure. Similar tests on mortar and cement had drying times with a standard deviation of 14.6 hours. If you want to be 98% confident that your experiment will have a mean drying time that is within 1.5 hours of the mean drying time for all possible samples of concrete, how many samples will you need to include in your experiment?

Sample size for determining a mean

$$n = \left[\frac{Z_{\alpha/2} \cdot \hat{\sigma}}{E}\right]^2$$

confidence = 0.98

$\alpha = 0.02$ in 2 tails

$Z_{\alpha/2} = 2.33$

$\hat{\sigma} = 14.6$ for mortar and cement

$E = 1.5$ within this acceptable error

$$= \left[\frac{(2.33)(14.6)}{1.5}\right]^2$$

$= 514.3$

515 samples to test

(6 points)

11. A grocery store needs to know how much beef to have available for sale. They want to estimate the average number of pounds of beef eaten per household each week in the neighborhood the store serves. The standard deviation (σ) for weekly beef consumption by household is about 4 pounds. If the store operators want to be 90% confident that the average beef consumption for their sample of houselholds is within 0.5 pound of μ, how many households should they include in their sample?

$$\boxed{\text{Sample size for estimating a mean}}$$

$$n = \left[\frac{z_{\alpha/2} \cdot \hat{\sigma}}{E} \right]^2$$

confidence = 0.90

$\alpha/2 = 0.05$

$z_{\alpha/2} = 1.645$

$$= \left[\frac{(1.645)(4)}{0.5} \right]^2$$

$\hat{\sigma} \cong 4$

$$= 19.2$$

$$\boxed{\text{20 households need to be sampled}}$$

$E = 0.5$
acceptable error

(6 points: 6 minutes)

12. ***Consumer Reports*** **magazine wants to know how long a new brand of "3-hour" fire logs really burn. Three years ago they tested other brands of fire logs and the times had a standard deviation of 15.2 minutes. If *Consumer Reports* wants to be 90% confident that the average burn time for their sample of new fire logs is within 2 minutes of the mean burn time for all of the new fire logs, how many logs should they randomly select and test?**

$\boxed{\text{sample size for estimating an average}}$

$$n = \left[\frac{z_{\alpha/2} \cdot \hat{\sigma}}{E}\right]^2$$

confidence = 0.90

$\alpha = 0.10$

$z_{\alpha/2} = 1.645$

$$= \left[\frac{(1.645)(15.2)}{2}\right]^2$$

$$= 156.3 \uparrow$$

$\hat{\sigma} = 15.2$
from similar tests 3 years ago.

157 fire logs should be tested.

$E = 2$
acceptable margin of error

A new gene has been discovered that makes people develop allergies. The Centers for Disease Control (CDC) wants to estimate the proportion of people that have this gene. About 20% of all people suffer from allergies. The CDC wants to do a study that will give them 98% confidence that their sample proportion is within 0.05 of the true proportion. How many people should the CDC include in their study?

$\boxed{\text{sample size for estimating a proportion}}$

$$n = \frac{(Z_{\alpha/2})^2 \hat{p}\hat{q}}{E^2}$$

confidence = 0.98

$\alpha = 0.02$

$Z_{\alpha/2} = 2.33$

$$= \frac{(2.33)^2(.20)(.80)}{(0.05)^2}$$

$$= 347.4 \uparrow$$

$\hat{p} = 0.20$ since about 20% of people suffer from allergies

$\hat{q} = 0.80$

348 people should be included in the study

$E = 0.05$ acceptable error

(6 points: 5 minutes)

proportion

3. The federal government wants to design a survey to estimate the fraction of people who want choices for handling Social Security taxes (no related information is available at this time). If the sample proportion of people who want Social Security choice must be within 3 percentage points of the population proportion with 95% confidence, how many people must be included in the random survey sample?

Sample size for estimating a proportion

$$n = \frac{(Z_{\alpha/2})^2 \hat{p} \hat{q}}{E^2}$$

confidence = 0.95

$\alpha = 0.05$

$Z_{\alpha/2} = 1.96$

$$= \frac{(1.96)^2 (.5)(.5)}{(0.03)^2}$$

$$= 1067.1 \uparrow$$

$\hat{p} =$ ~~[illegible]~~ 0.50

$\hat{q} = 0.50$

because No information to estimate p is available

1068 people should be randomly selected for the survey

$E = 0.03$

acceptable margin of error

(8 points - 8 minutes)

3. **Two flea-killing products are tested. To reduce the variation in the test, both products are used on each dog in the experiment. Use the data to construct a 98% confidence interval for the true difference in average number of fleas ($\mu_1 - \mu_2$) left by the two products.**

$\mu_d = (\mu_1 - \mu_2)$

Number of Fleas

	Product	
Dog	**1**	**2**
1	1	7
2	35	41
3	24	22
4	13	13
5	1	7

$d = x_1 - x_2$

d
-6
-6
2
0
-6

$n = 5$

$\bar{d} = -3.2$

$S_d = 3.899$

$df = 4$

confidence $= 0.98$

$\alpha = 0.02$ in 2 tails

$t_{\alpha/2} = 3.747$

$98\%\ CI(\mu_d) =$

$\bar{d} \pm t_{\alpha/2}\left(\frac{S_d}{\sqrt{n}}\right)$

$= (-3.2) \pm 3.747\left(\frac{3.899}{\sqrt{5}}\right)$

$= -3.2 \pm (3.747)(1.7437)$

$= -3.2 \pm 6.53$

$= \left[-9.73 < (\mu_1 - \mu_2) < 3.33\right]$

(10 points - 10 minutes)

3. A medical clinic examines 6 patients to test the clain that vigorous exercise for 10 minutes will increase a person's pulse (heart) rate by more than 30 beats per minute. Use the data from the experiment to test whether the mean pulse rate after exercise (μ_2) is more than 30 beats faster per minute than the mean pulse rate before exercise (μ_1).

(use α = 0.01)

	Pulse Rate		$(X_A - X_B)$
Patient	Before Exercise	After Exercise	d
1	58	85	27
2	66	92	26
3	67	94	27
4	64	86	22
5	64	90	26
6	57	80	23

claim:

$\mu_{\text{Exercise}} > \mu_{\text{RESTING}} + 30$

Exercise: ~~Before~~ after; RESTING: Before exercise

$(\mu_A - \mu_B) > 30$

$\mu_d > 30$

$H_0: (\mu_A - \mu_B) \le 30 \quad (\mu_d \le 30)$

$H_1: (\mu_A - \mu_B) > 30 \quad (\mu_d > 30)$

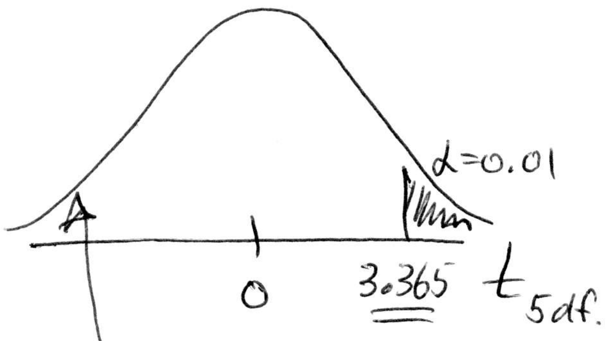
$\alpha = 0.01$ right tail

$n = 6 \quad df = 5$

$\bar{d} = 25.17$

$s_d = 2.137$

critical region



Test Statistic

$$\frac{\bar{d} - \mu_d}{s_d/\sqrt{n}}$$

$$= \frac{25.17 - 30}{2.137/\sqrt{6}}$$

$$= \frac{-4.83}{0.872}$$

$$= -5.539$$

Conclusion:

Do not reject H_0:

(10 points : 15 minutes)

5. **You run a company that produces cans of mixed nuts labled "400 grams". A requirement of the federal government is that the moisture content of the nuts (as a group) cannot be more than four percent (4 grams water per 100 grams of nuts). You have two different ways to measure the moisture content, called Method 1 and Method 2. Use the data below for 8 samples of nuts to make a 95% confidence interval for (μ_1 - μ_2), the difference between the mean for Method 1 and the mean for Method 2.** (You must include the algebraic expression for the CI in your answer.)

Sample	Moisture (grams) Measured by Method 1	Method 2	$(x_1 - x_2)$ d
1	20.9	19.8	1.1
2	19.4	23.2	-3.8
3	12.7	12.4	0.3
4	18.8	22.1	-3.3
5	10.8	14.6	-3.8
6	16.8	17.9	-1.1
7	16.3	17.6	-1.3
8	13.6	15.3	-1.7
$\bar{x}$ =	16.16	17.86	
s =	9.53	3.73	

$n = 8 \quad df = 7$

$\bar{d} = -1.7$

$s_d = 1.842$

$\alpha = 0.05$ 2 tails

$t_{\alpha/2} = 2.365$

$95\% \ CI(\mu_1 - \mu_2) =$

$95\% \ CI(\mu_d) =$

$\bar{d} \pm t_{\alpha/2}\left(\frac{s_d}{\sqrt{n}}\right)$

$= (-1.7) \pm 2.365\left(\frac{1.842}{\sqrt{8}}\right)$

$= (-1.7) \pm 1.54$

$= [-3.24 < (\mu_1 - \mu_2) < -0.16]$

(8 points : 12 minutes)

6. **You run a company that produces cans of mixed nuts labled "400 grams". A requirement of the federal government is that the moisture content of the nuts (as a group) cannot be more than four percent (4 grams water per 100 grams of nuts). You have two different ways to measure the moisture content, called Method 1 and Method 2. Use the data given in problem #5 to test the claim that μ_2 is at least 0.5 grams more than μ_1.** (Use a 0.10 significance level for this test.) (You must include the algebraic expression for the test statistic as part of your answer.)

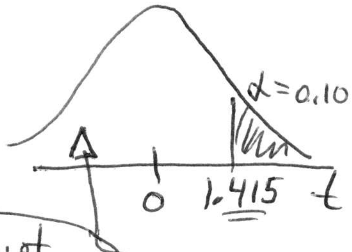
claim: $\mu_2 \geq \mu_1 + 0.5 \qquad (-0.5) \geq (\mu_1 - \mu_2)$ OR $(\mu_1 - \mu_2) \leq (-0.5)$

$H_0: (\mu_1 - \mu_2) \leq -0.5$

$H_1: (\mu_1 - \mu_2) > -0.5$

$\alpha = 0.10$ right tail

$df = 7 \quad t = 1.415$



$\frac{\bar{d} - \mu_d}{s/\sqrt{n}} = \frac{(-1.7) - (-0.5)}{1.842/\sqrt{8}}$

$= \frac{-1.2}{0.65} = -1.846$

Do not reject H_0!

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