Statistics 300: Elementary Statistics

Section 9-2

Section 9-2 concerns Confidence Intervals and Hypothesis tests for the difference of two proportions, $(p_1 - p_2)$

What is the variance of

$$(\hat{p}_1 - \hat{p}_2)\,?$$

- **New concept:**

If x and y are independent random variables, then the variance of their difference is the sum of their variances

$$s^2_{x-y} = s^2_x + s^2_y$$

What is the variance of

$$(\hat{p}_1 - \hat{p}_2)\,?$$

- **Apply the new concept (step 1):**

$$x = \hat{p}_1$$
$$y = \hat{p}_2$$
$$s^2_{\hat{p}_1 - \hat{p}_2} = s^2_{\hat{p}_1} + s^2_{\hat{p}_2}$$

What is the variance of

$$(\hat{p}_1 - \hat{p}_2)\,?$$

- **Apply the new concept (step 2):**

$$s^2_{\hat{p}_1} = \frac{p_1 q_1}{n_1}$$
$$s^2_{\hat{p}_2} = \frac{p_2 q_2}{n_2}$$
$$s^2_{\hat{p}_1 - \hat{p}_2} = \frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}$$

What is the variance of

$$(\hat{p}_1 - \hat{p}_2)\,?$$

- **Apply the new concept (step 3):**

replace the unknown p and q with their sample estimates

$$s^2_{\hat{p}_1 - \hat{p}_2} = \frac{\hat{p}_1 \hat{q}_1}{n_1} + \frac{\hat{p}_2 \hat{q}_2}{n_2}$$

The variance of $(\hat{p}_1 - \hat{p}_2)$

Is used in the CI($p_1 - p_2$ **)**

$$CI(p_1 - p_2) = (\hat{p}_1 - \hat{p}_2) \pm Z_{\alpha/2}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}$$

Tests concerning (p_1 - p_2)
Test Statistic

$$\frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)_0}{S_{(p_1 - p_2)}}$$

The denominator has two forms depending on H_0:

If H_0: relates ($p_1 - p_2$) to zero, then use this test statistic:

$$\frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)_0}{\sqrt{\frac{\bar{p}\bar{q}}{n_1} + \frac{\bar{p}\bar{q}}{n_2}}}$$

$$\text{where } \bar{p} = \frac{x_1 + x_2}{n_1 + n_2}$$

If H_0: relates $(p_1 - p_2)$ to a non-zero value, then use this test statistic:

$$\frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)_0}{\sqrt{\frac{\hat{p}\hat{q}}{n_1} + \frac{\hat{p}\hat{q}}{n_2}}}$$

Do not use $\bar{p}$

Section 9-2
Handling Claims / Hypotheses

- **Write the claim in a symbolic expression as naturally as you can**
- **Then rearrange the expression to have the difference between the two means on one side of the relational operator (< > = …)**

Section 9-2
Handling Claims / Hypotheses

- **Statement: proportion #1 is less than 0.04 more than proportion #2**
- **So** $p_2 < p_1 + 0.04$
- **Rearrange** $(p_2 - p_1) < 0.04$
- **H_0:** $(p_2 - p_1) \geq 0.04$
- **H_1:** $(p_2 - p_1) < 0.04$

Section 9-2
Handling Claims / Hypotheses

- **Statement: Proportion A is 0.18 more than proportion B**
- **So:** $p_A = p_B + 0.18$
- **Rearrange:** $(p_A - p_B) = 0.18$
- **H_0:** $(p_A - p_B) = 0.18$
- **H_1:** $(p_A - p_B) \neq 18$