

(3 points, 2 minutes)

1. What are the appropriate expressions (formulas) for the mean, variance, and standard deviation in the context of a discrete probability distribution?

Parameter	Expression
$\mu$	$\sum x \cdot P(x)$
$\sigma^2$	$\sum (x - \mu)^2 \cdot P(x)$
$\sigma$	$\sqrt{\sum (x - \mu)^2 \cdot P(x)}$

(7 points : 7 minutes)

2. For each of the problems below (here and on next page), determine whether a valid probability distribution is described and, if so, calculate the mean, variance, and standard deviation.

(a) A manufacturer makes a product that can have defects in 5 different ways, so some of the product will have 0 defects, some 1, 2, 3, 4, or 5 defects. The manufacturer claims that 82.8% have 0 defects, 12.8% have 1, 1.3% have 2, 0.9% have 3, 0.2% have 4, and 0.1% have 5 defects.

$\mu =$  \_\_\_\_\_  
 $\sigma^2 =$  \_\_\_\_\_  
 $\sigma =$  \_\_\_\_\_

x	P(x)				
0	0.828				
1	0.128				
2	0.013				
3	0.009				
4	0.002				
5	0.001				

$\sum P(x) = 0.981$

*Not a valid distribution*

2. (Continued)

- (b) Use the relative frequency in the recent past as a probability distribution for the near future. Judges in an ice skating contest award scores from 1 through 6 for 288 contestants. Ten judges awarded a total of 2880 scores in the proportions listed as probabilities in the distribution below. What are the mean, variance and standard deviation of this distribution?

$$\mu = \underline{3.48}$$

$$\sigma^2 = \underline{2.189}$$

$$\sigma = \underline{1.48}$$

x	P(x)	x · P(x)	$(x - \mu)^2 \cdot P(x)$
1	0.11	0.11	0.677
2	0.17	0.34	0.372
3	0.21	0.63	0.048
4	0.26	1.04	0.070
5	0.14	0.70	0.323
6	0.11	0.66	0.699

$$\sum x \cdot P(x) = \mu = 3.48$$

$$\sum (x - \mu)^2 \cdot P(x) = 2.189 = \sigma^2$$

$$\sigma = \sqrt{\sigma^2} = 1.48$$

(5 points : 4 minutes)

3. Larry is sitting in an airport waiting for his son's flight, which will arrive in 8 hours. Larry is bored. He will get 12 M&Ms from a candy machine. The proportion of "RED" M&Ms among all the M&Ms made is 31%. If Larry's 12 M&Ms are a random selection from the population of all M&Ms, what is the probability that his 12 M&Ms will include exactly 5 RED candies?

$p = P(\text{red}) = 0.31$      $P(\text{Not Red}) = q = 0.69$      $n = 12$      $x = 5$   
 random and small sample from large population implies  
 "independence" and  $p$  stays the same from one M&M to  
 the next.    Binomial     $X \sim B(12, 0.31)$

$$P(X=5) = {}_{12}C_5 (0.31)^5 (0.69)^7 = \underline{0.169}$$

(9 points : 7 minutes)

4. (a) The percentage of "RED" M&Ms among all the M&Ms that are made is 31%. What are the mean and standard deviation for the number of RED M&Ms in randomly selected samples of 1800 M&Ms?

Binomial:  $n = 1800$        $\mu = np = 558$   
 $p = 0.31$        $\sigma = \sqrt{npq} = 19.6$

- (b) In a random sample of 1800 M&Ms, would it be unusual to find more than 580 RED M&Ms?

$X = 580$   
 $\mu = 558$   
 $\sigma = 19.6$  } from part (a)

$$Z = \frac{X - \mu}{\sigma} = \frac{580 - 558}{19.6} = \underline{\underline{1.12}}$$

↑  
Not unusual