Statistics 300 Tue./Thu.
Quiz #16 7:00-8:50 p.m. Name: Solution

(8 points : 12 minutes)

1. **A random sample of 13 Zoologists has an average weight of 106 kg with a standard deviation of 22 kg. A random sample of 17 Physicists has an average weight of 100 kg and a standard deviation of 24 kg. Use these results to construct a 95% confidence interval for the difference between the mean weight of all zoologists and the mean weight of all physicists.** (Assume that variation among weights is the same in both cases.) $(\sigma_Z^2 = \sigma_P^2)$
(You must include the algebraic expression for the test statistic as part of your answer.)

sample data		
	Zoologists	**Physicists**
n =	13	17
$\bar{X}$ =	106	100
s =	22	24

$$95\%\ CI(\mu_Z - \mu_P) = (\bar{x}_Z - \bar{x}_P) \pm t_{\alpha/2}\sqrt{\frac{S^2_{Pool}}{n_Z} + \frac{S^2_{Pool}}{n}}$$

$$= (106-100) \pm 2.048\sqrt{\frac{536.6}{13} + \frac{536.6}{17}}$$

$$= 6 \pm 17.5$$

$$= [-11.5 < \mu_Z - \mu_P < 23.5]$$

pool variances and add degrees of freedom

$$S^2_{Pool} = \frac{(n_Z-1)S_Z^2 + (n_P-1)S_P^2}{(n_Z-1)+(n_P-1)} = \frac{(12)(22)^2 + (16)(24)^2}{12+16}$$

$$S^2_{Pool} = 536.6$$

$df = 12 + 16 = 28$

$\alpha = 1 -$ confidence $= 1 - 0.95 = 0.05$ in 2 tails

$t_{\alpha/2} = 2.048$ df=28

(8 points : 12 minutes)

2. **A random sample of 16 Zoologists has an average weight of 106 kg with a standard deviation of 28 kg. A random sample of 10 Physicists has an average weight of 100 kg and a standard deviation of 20 kg. Use these results to test the claim that the mean weight of all zoologists is more than 2 kg greater than the mean weight of all physicists.**
(Assume that variation among the weights in each population may not be the same.) $(\sigma_Z^2 \neq \sigma_P^2)$
(You must include the algebraic expression for the test statistic as part of your answer.)

$(\alpha = 0.05)$

sample data		
	Zoologists	**Physicists**
n =	16	10
$\bar{X}$ =	106	100
s =	28	20
df	15	9

claim: $\mu_Z > \mu_P + 2$

$(\mu_Z - \mu_P) > 2$

$\alpha = 0.05$ right tail.

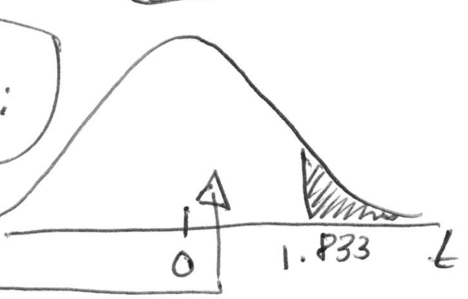
so ① do not pool variances and ② use the smaller sample's deg. of freedom

$t = 1.833$

$$\frac{(\bar{x}_Z - \bar{x}_P) - (\mu_Z - \mu_P)_0}{\sqrt{\frac{S_Z^2}{n_Z} + \frac{S_P^2}{n_P}}}$$

$$= \frac{(106-100) - 2}{\sqrt{\frac{28^2}{16} + \frac{20^2}{10}}} = \frac{4}{9.43} = 0.42$$

Do Not reject H_0:



Statistics 300 Tue./Thu.
Quiz #16 7:00-8:50 p.m. **Name:** Solution

(6 points : 10 minutes)

3. **Some lawyers argue that police radar units are too variable to give reliable speed values. Police laboratories test a new radar unit that is claimed to have lower variability, and they compare its performance with an old unit. Use the test data to test the claim that the variability of the new radar unit is less than the variability of the old one. The readings from both units are normally distributed.**
(Use a Type I error probability of 0.025.)

sample data		
	New Unit	**Old Unit**
n =	10	8
$\overline{x}$ =	68.4	68.3
s =	0.22	0.27

Claim: $\sigma^2_{New} < \sigma^2_{OLD}$

H0: $\sigma^2_{New} \geq \sigma^2_{OLD}$

H1: $\sigma^2_{New} < \sigma^2_{OLD}$

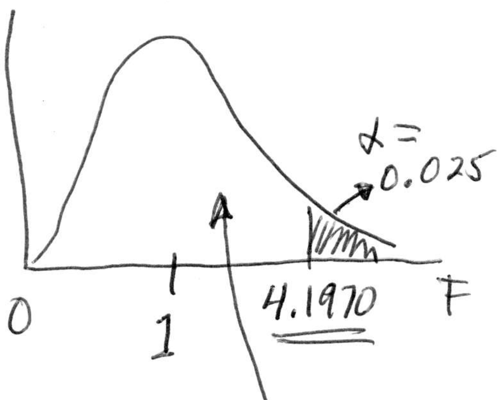
Put S^2_{New} in denominator of test statistic

put S^2_{OLD} in Numerator of test statistic

$$F = \frac{S^2_{OLD}}{S^2_{New}}$$

$$= \frac{(0.27)^2}{(0.22)^2} \quad \frac{7 \text{ d.f.}}{9 \text{ d.f.}}$$

$$= 1.5062$$



Do not reject Ho ;

Statistics 300 Tue./Thu.
Quiz #16 7:00-8:50 p.m. Name: Solution

(8 points : 12 minutes)

4. **You run a company that produces cans of mixed nuts labled "400 grams". A requirement of the federal government is that the moisture content of the nuts (as a group) cannot be more than four percent (4 grams water per 100 grams of nuts). You have two different ways to measure the moisture content, called Method 1 and Method 2. Use the data below for 16 samples of nuts to make a 95% confidence interval for (μ_1 - μ_2), the difference between the mean for Method 1 and the mean for Method 2.**
(You must include the algebraic expression for the CI in your answer.)

Looks like matched pairs, but it is Not. There were 16 samples, 8 for Method 1 and another (unmatched) set of 8 for Method 2.

I forgot to say to consider $\sigma_1^2 = \sigma_2^2$.

pool variances and add deg. of freedom.

	Moisture (grams)	
Sample	Method 1	Method 2
1	21.4	19.8
2	23.6	23.2
3	12.6	12.4
4	22.9	22.1
5	16.0	14.6
6	19.2	17.9
7	17.9	17.6
8	16.1	15.3
$\bar{x}$ =	18.71	17.86
s =	3.80	3.73

df = 7 + 7 = 14

$$\boxed{S_p^2 = \frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{(n_1-1)+(n_2-1)} = \frac{(7)(3.80)^2 + (7)(3.73)^2}{7+7} = 14.18}$$

$t_{\alpha/2} = 2.145$ 14 d.f.

$$95\%\ CI(\mu_1 - \mu_2) = (\bar{X}_1 - \bar{X}_2) \pm t_{\alpha/2}\sqrt{\frac{S_p^2}{n_1} + \frac{S_p^2}{n_2}}$$

$$= (18.71 - 17.86) \pm 2.145\sqrt{\frac{14.18}{8} + \frac{14.18}{8}}$$

$$= 0.85 \pm (2.145)(1.8828)$$

$$= 0.85 \pm 4.039$$

$$[-3.189 < (\mu_1 - \mu_2) < 1.254]$$

Statistics 300 Tue./Thu.
Quiz #16 7:00-8:50 p.m. Name: Solution

(8 points : 12 minutes)

5. You run a company that produces cans of mixed nuts labled "400 grams". A requirement of the federal government is that the moisture content of the nuts (as a group) cannot be more than four percent (4 grams water per 100 grams of nuts). You have two different ways to measure the moisture content, called Method 1 and Method 2. Use the data below to test the claim that μ_2 is at least 0.5 grams more than μ_1. (Let α = 0.10 and assume the variances for the methods are not the same. You must include the algebraic expression for the test statistic as part of your answer.) $(\sigma_1^2 \neq \sigma_2^2)$

so do not pool variances and only use the smallest of the two degrees of freedom.

	Moisture (grams)	
	Method 1	**Method 2**
	20.7	19.8
	23.8	23.2
	13.0	12.4
	24.2	22.1
	15.2	14.6
	19.6	17.9
	18.6	17.6
	15.4	15.3
		18.6
		19.3
$\bar{x}$ =	18.81	18.08
s =	4.08	3.32
	$n = 8$	$n = 10$
	$df = 7$	$df = 9$

claim: $\mu_2 \geq \mu_1 + 0.5$

$(\mu_2 - \mu_1) \geq 0.5$

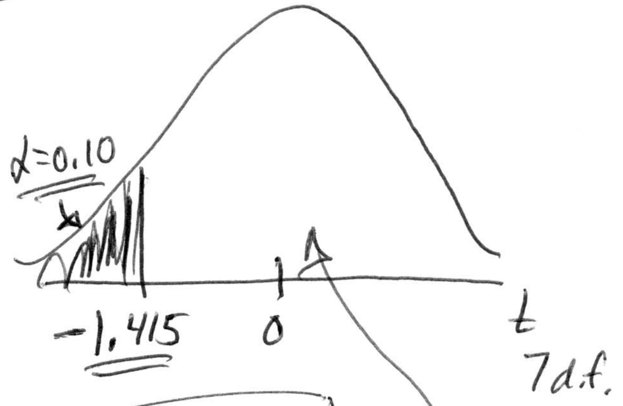
$H_0: (\mu_2 - \mu_1) \geq 0.5$

$H_1: (\mu_2 - \mu_1) < 0.5$

$\alpha = 0.10$ in left tail

$$\frac{(\bar{X}_2 - \bar{X}_1) - (\mu_2 - \mu_1)_0}{\sqrt{\frac{S_2^2}{n_2} + \frac{S_1^2}{n_1}}}$$

$$= \frac{(18.81 - 18.08) - 0.5}{\sqrt{\frac{(4.08)^2}{8} + \frac{(3.32)^2}{10}}} = \frac{0.73 - 0.5}{1.784} = \frac{0.23}{1.784} = 0.129$$



Do Not reject H_0!