3/13

$\hat{p}$ = sample proportion = random var.

p = mean of all possible sample proportion or population proportion

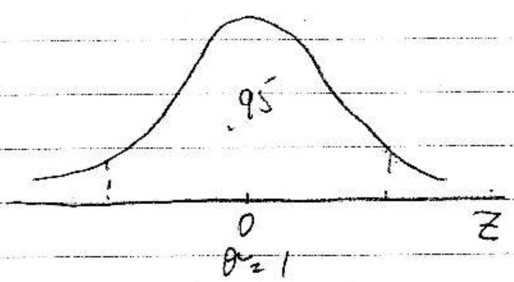
x has mean μ

Standart dev. σ

$\bar{x} \rightarrow \mu$

$$\sigma_{\hat{p}} = \sqrt{\frac{pq}{n}}$$

$$\frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} \sim N(0,1)$$



$$\begin{array}{r} \text{random var.} \\ \text{mean} \\ \text{st. dev} \end{array} \begin{pmatrix} x \\ \mu \\ \sigma \end{pmatrix}$$

samples of size n

$$\begin{pmatrix} \bar{x} \\ \mu_{\bar{x}} \\ \sigma_{\bar{x}} \end{pmatrix} = \begin{pmatrix} \bar{x} \\ \mu \\ \sigma/\sqrt{n} \end{pmatrix} \begin{array}{l} \text{ran var} \\ \text{mean} \\ \end{array}$$

$$\frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \quad \left\{ \begin{array}{l} \text{if } x \sim N(\mu, \sigma) \\ \text{or } n > 30 \end{array} \right\} \Rightarrow$$

$$\Rightarrow \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \sim N(0,1)$$

$$CI(p) = \hat{p} \pm Z_{\alpha/2} \sqrt{\frac{pq}{n}} \Rightarrow \hat{p} \pm Z_{\alpha/2} \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

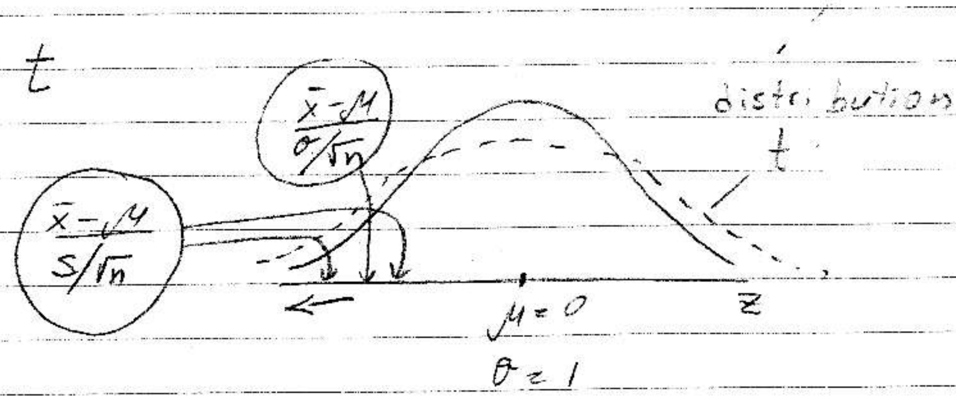
$$CI(\mu) = \bar{x} \pm Z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right) \Rightarrow \bar{x} \pm Z_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right)$$

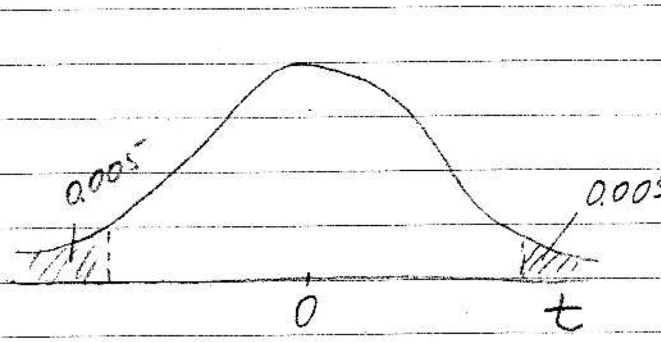
confidence

data

$$\frac{\bar{x} - \mu}{s/\sqrt{n}} \sim t$$

p. 333





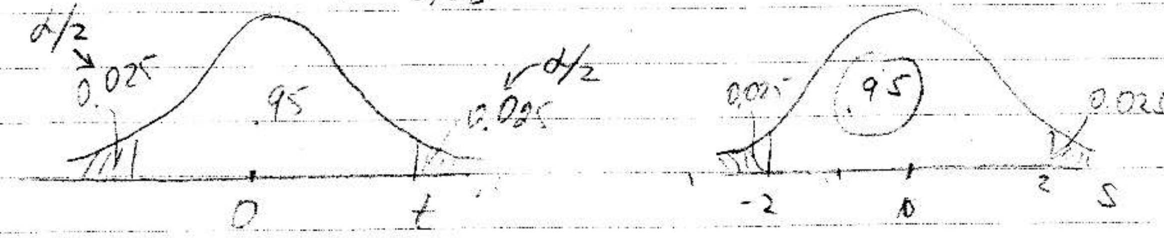
10
?
30

$\bar{x} = 20$ $n = 3$

$\bar{x} = \frac{\sum x}{3}$ $3\bar{x} = \sum x$

Degrees of Freedom = $(n-1)$

$\alpha = 0.05$



$\underline{\text{NOT}}$ $CI(\mu) = \bar{x} \pm Z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$

$\underline{\text{this one}}$ $CI(\mu) = \bar{x} \pm t_{\alpha/2}\left(\frac{S}{\sqrt{n}}\right)$

proportion use Z $\hat{p}$ use Z
about μ use t μ use t

$Z = 1.645$

Ex

n	d.f. degrees of freedom	confidence	in 2 tails α	in 1 tail $\alpha/2$	t
20	19	0.90	0.10	0.05	1.729
17	16	0.95	0.05	0.025	2.120

ROBUST

Even when the original distribution of "x" values is not normal, still this confidence interval is usually pretty good

7-3 Sample Size needed for an experiment about μ

7-4 $\bar{X} \pm t_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right)$

$\bar{x} \pm E$

$$E = Z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right) \qquad \bar{x} \pm Z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$$

confidence

estimate (a guess) about σ

$$n = \left[\frac{Z_{\alpha/2} \cdot \sigma}{E}\right]^2$$

$|\bar{x} - \mu|$

$CI(\sigma)$

p use Z
μ use t
σ use χ^2 chi square

$$\frac{(n-1)s^2}{\sigma^2} \sim \chi^2_{(n-1)d.f.} \qquad \frac{x-\mu}{s/\sqrt{n}} \sim t_{(n-1)d.f.}$$

$$\frac{(n-1)\frac{\Sigma(x-\bar{x})^2}{(n-1)}}{\sigma^2} \qquad s^2 = \frac{\Sigma(x-\bar{x})^2}{n-1}$$

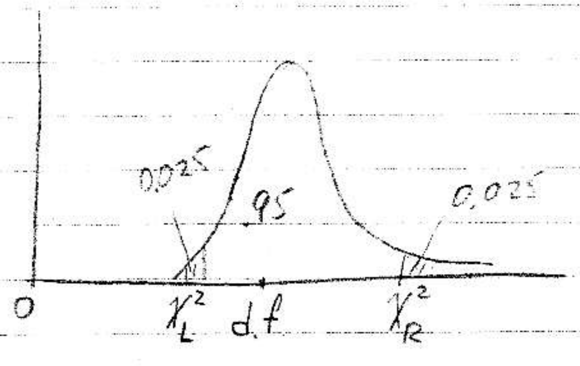
$$\frac{\sum_{i=1}^{n}(x-\bar{x})^2}{\sigma^2} = \frac{1}{\sigma^2}\left[(x_1-\bar{x})^2 + (x_2-\bar{x})^2 + \dots + (x_n-\bar{x})^2\right]$$

$$= \frac{(x_1-\bar{x})^2}{\sigma^2} + \frac{(x_2-\bar{x})^2}{\sigma^2} + \dots + \frac{(x_n-\bar{x})^2}{\sigma^2} = \longrightarrow$$

$$= \left(\frac{x_1-\bar{x}}{\sigma}\right)^2 + \left(\frac{x_2-\bar{x}}{\sigma}\right)^2 + \cdots + \left(\frac{x_n-\bar{x}}{\sigma}\right)^2$$

Z

p.346



$$P\left[\chi_L^2 < \frac{(n-1)S^2}{\sigma^2} < \chi_R\right] = 0.95$$

$$\frac{(n-1)s^2}{\chi_R^2} < \sigma^2 < \frac{(n-1)S^2}{\chi_L^2}$$

Ex 1. 90% $CI(\sigma^2)$

$n = 14$

$d.f = 13$

$\chi_L^2 = 5.892$

$\chi_R^2 = 22.362$

Ex 2. Confid = 0.95

$n = 20$ $\quad d.f = 19$ $\quad \alpha = 0.05$ $\quad \alpha/2 = 0.025$

$\chi_L^2 = 8.907$

$\chi_R^2 = 32.852$

Ex. 3. $N = 4$

$p(\text{Black}) = 0.9$ $\quad$ 900,000

$p(\text{Red}) = 0.1$ $\quad$ 100,000

$P(3 \text{ or } 4 \text{ red})$

$${}_4C_4(0.1)^4(0.9)^0 + {}_4C_3(0.1)^3(0.9)^1 = 0.0001 + 0.0036 = 0.0037$$

X = 3 red and 1 black

Risk:

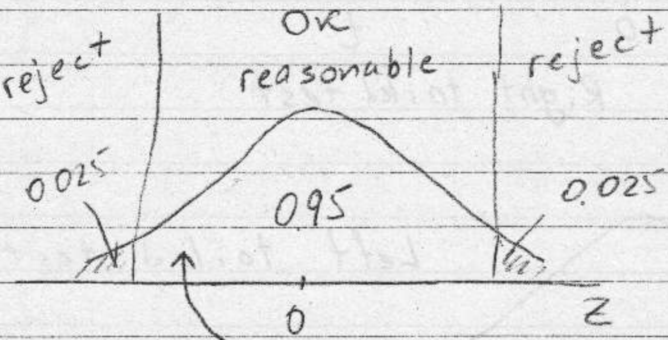
What P(rejecting Hypothesis | TRUE)?

reasonable 0.10 = α

ok? 0.05

not so much? 0.01

not good 0.001 unusual



$5\% = \alpha$

Hypo = 0.42

$$\frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}}$$

μ

p

σ or σ^2

$H_0: \mu \le 20$ must include (=)

$H_1: \mu > 20$ can't have (=)

$H_0: \mu \ge 20$

$H_1: \mu < 20$ Left tailed test

$H_0: \mu = 20$

$H_1: \mu \ne 20$ two tailed test

Test Statistic

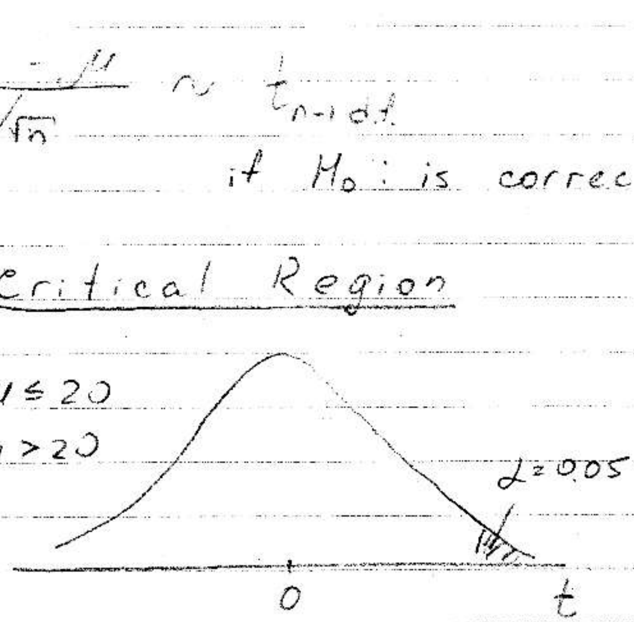
$$\frac{\bar{x}-\mu}{s/\sqrt{n}} \sim t_{n-1\,d.f.}$$

if H_0: is correct

$H_0: \mu \le 20$
$H_1: \mu > 20$

Critical Region

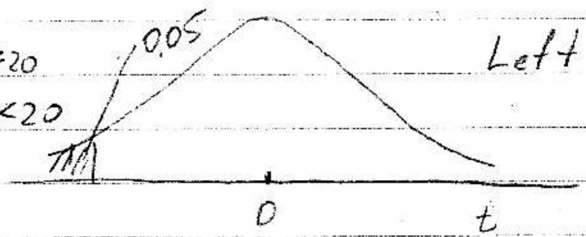
$H_0: \mu \le 20$
$H_1: \mu > 20$



$$\frac{\bar{x}-20}{s/\sqrt{n}}$$

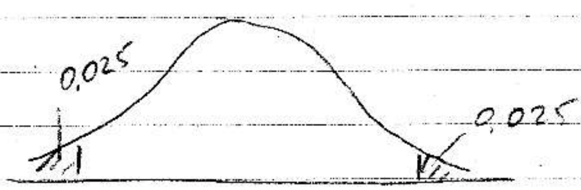
Right tailed test

$H_0: \mu \ge 20$
$H_1: \mu < 20$



Left tailed test

$H_0: \mu = 20$
$H_1: \mu \ne 20$



Two tailed test

Conclusion