3/6 Continuous Prob. Distributions

- Uniform $x \sim U[a, b]$ $x \to$ Prob.
  Prob $\to x$

- Standart Normal $Z \sim N(\mu = 0, \theta = 1)$
  $Z \to$ Prob or Prob $\to Z$

- Normal Distrib. $x \to Z$ and $Z \to$ Prob
  $P_k \to Z_k \to x = P_k$
  $x \sim N(\mu, \theta)$

- CLT $\bar{X}$

$$\left.\begin{matrix} \bar{X} \\ \mu \\ \theta \\ n \end{matrix}\right\} \to \begin{matrix} \bar{X} \\ \mu_{\bar{x}} = \mu \\ \theta_{\bar{x}} = \theta/\sqrt{n} \end{matrix}$$

$\bar{x} \to Z \to$ Prob.

$$Z = \frac{x - \mu}{\theta} = \frac{\bar{x} - \mu_{\bar{x}}}{\theta_{\bar{x}}} \text{ or } \frac{\bar{x} - \mu}{\theta/\sqrt{n}}$$

Prob $\to Z \to \bar{x}$ sample average

1) $X \sim N(100, 8)$
sample of 12 items
Find 80^{th} piocentile of the Dist. of all possible $\bar{X}$.
$n = 12$
$\bar{x} \sim N\left(100, \frac{8}{\sqrt{12}}\right)$
$k = 0.80$
$Z_{80} = 0.85$
$\bar{x}_{80} = P_{80} = Z_{80} \cdot \theta_{\bar{x}} + \mu_{\bar{x}}$

	0.05
0.8	0.80

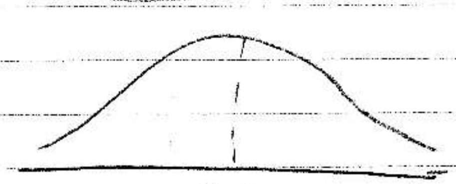
Ex Quiz 9(1)

	μ	σ	n	$\mu_{\bar{x}}$	$\sigma_{\bar{x}}$
a	216	27	81	216	3
b	200	27	9	200	9
c	216	27	81	216	3
d	200	27	9	200	9
e	88	27	9	88	3
f	1023	27	3	1023	9
g	500	43	15	500	11.103

$\mu = \mu_{\bar{x}}$ $\qquad$ $\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}}$

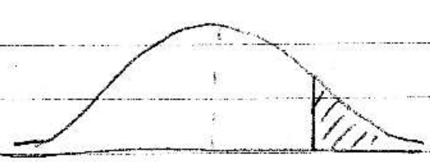
Ex Pig weights are normaly dist.
$\mu = 140$ lbs $\quad \sigma = 10$

$P(145 < \bar{x})$ - ?
$n = 18$



140 $\quad X = w$

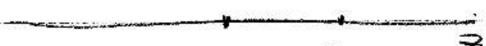
$\sigma = 10$



140 $\quad$ 145 $\quad \bar{x}$

$\sigma_{\bar{x}} = \dfrac{10}{\sqrt{18}} = 2.36$

$Z = \dfrac{145-140}{2.36} = 2.12$



0 $\quad$ 2.12 $\quad$ z

	0.02
-2.1	$\boxed{0.0170}$

P = 0.0170

Ex

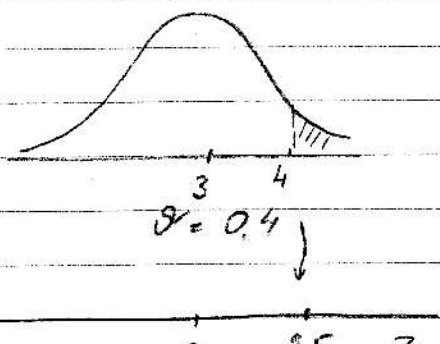
1000 animals

$_{1000}C_{10} = 2.63 \times 10^{23}$

p 277 # 13. $\mu = 3 \quad \sigma = 0.40$

time $x \sim N(3, 0.4)$

a) P(a random x > 4)



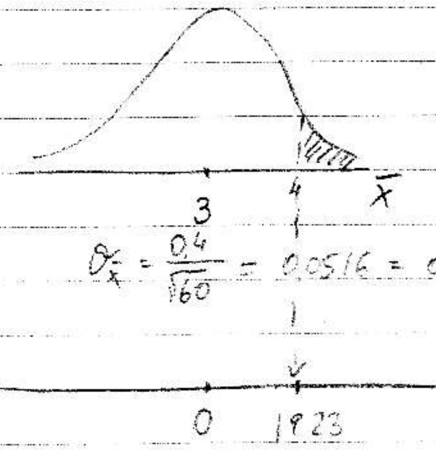
$$z = \frac{x-\mu}{\sigma} = \frac{4-3}{0.4} = \frac{1}{0.4} = 2.5$$

0.00

$-2.5 \quad \boxed{0.0062}$

b) n = 60 $\quad P(\bar{x} > 4)$

random sample of 60



$\sigma_{\bar{x}} = \frac{0.4}{\sqrt{60}} = 0.0516 = 0.052$

$$z = \frac{\bar{x}-\mu}{\sigma_{\bar{x}}} = \frac{4-3}{0.052} =$$

$= 19.23$

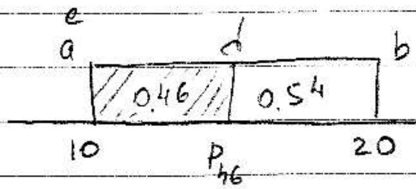
almost none

0.0001

$\bar{x}_{80} = P_{80} = 0{,}85 \cdot (2{,}31) + 100 = 101{,}9635$

Section 7-2

2) Find P_{46} - ? $\quad x \sim U[10, 20]$



$P_{46} =$

$$\frac{P_{46} - c}{b - a} = \frac{P_{46} - 10}{20 - 10} = 0{,}46$$

$$P_{46} = (0{,}46)(10) + 10 = 4{,}6 + 10 = 14{,}6$$

$\bar{x}$ $\quad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$ - two points in all problems

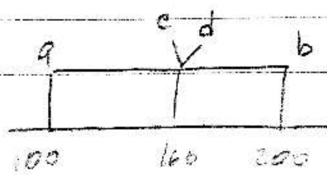
ch 7-2

We want to learn about

μ - population mean

$\mu \longleftrightarrow (\bar{x})$ point estimate

$\boxed{n = 1000}$ randomly sel.



$P = \frac{d - c}{b - a} = \frac{0}{100} = 0$

$\boxed{n = 2}$ less

interval estimate

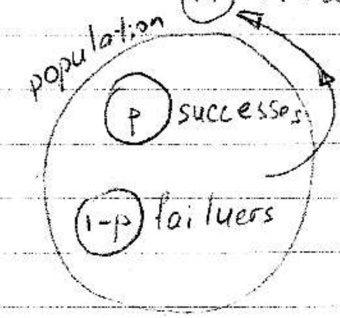
$[? - \bar{x} + ?] = \text{Prob} = 0{,}99$

μ

confidence interval
$CI(\mu)$
$CI(p)$ proportion = p
$\begin{cases} CI(\sigma) \\ CI(\sigma^2) \end{cases}$

1) Pop proportion : p
Sample proportion : $\hat{p}$

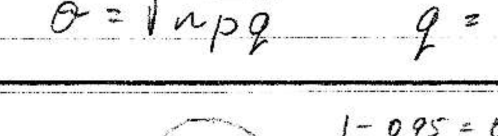
$$\hat{p} = \frac{x \text{ successes}}{n \text{ trials}}$$



If Binomial condition are true:
n = numbers of trials
p = Prob (successes)
$\mu = np$ x = total number of successes

$\sigma = \sqrt{npq}$ $q = 1-p$

$1 - 0.95 = 0.05 = \alpha$



$p = \mu_{\hat{p}}$

$\sigma_{\hat{p}} = \sqrt{\frac{pq}{n}}$ $Z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \frac{x - \mu}{\sigma}$

$-1.96 = Z$ $Z = 1.95$

0.06
1.9 →

$$P\left[-1.96 < \frac{\hat{p}-p}{\sqrt{\frac{pq}{n}}} < 1.96\right] = 0.95$$

$$P\left[-1.96\sqrt{\frac{pq}{n}} < \hat{p}-p < 1.96\sqrt{\frac{pq}{n}}\right] = 0.95$$

$$P\left[-\hat{p}-1.96\sqrt{\frac{pq}{n}} < -p < -\hat{p}+1.96\sqrt{\frac{pq}{n}}\right] = 0.95$$

$$P\left[\hat{p}+1.96\sqrt{\frac{pq}{n}} > p > \hat{p}-1.96\sqrt{\frac{pq}{n}}\right] = 0.95$$

$$P\left[\hat{p}-\underbrace{1.96}_{z_{\alpha/2}}\sqrt{\frac{\hat{p}\hat{q}}{n}} < \boxed{p} < \hat{p}+\underbrace{1.96}_{z_{\alpha/2}}\sqrt{\frac{\hat{p}\hat{q}}{n}}\right] = 0.95$$

Ex 1

Confidence	α	$\alpha/2$	$z_{\alpha/2}$
0.95	0.05	0.025	1.96
0.90	0.10	0.05	1.645
0.80	0.20	0.10	1.28
0.98	0.02	0.01	2.33
0.99	0.01	0.005	2.575

	0.04	0.05
-1.6		$\boxed{0.0504}$
	0.0495	0.0505

	0.08
-1.2	$\boxed{0.1003}$

	0.03
-2.3	$\boxed{.0099}$

$z_{\alpha/2} \rightarrow$ critical value

$z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} \rightarrow E$ or "margin of error"

$$CI(p) = \hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

Probability & Confidence

Probability is like a relative frequency that has not happened yet.

Ex $n = 884$ $\hat{p} = 0.13$ $\hat{q} = 0.87$ 98%

$Z_{\alpha/2} = 2.33$

Confid = 98%

$\alpha = 0.02$

$\alpha/2 = 0.01$

$Z_{\alpha/2} = 2.33$

$$98\%\ CI(p) = \hat{p} \pm Z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} =$$

$$= 0.13 \pm 2.33\sqrt{\frac{0.13 \cdot (0.87)}{884}} = 0.13 \pm 0.0221$$

$$[0.1079 < p < 0.1521]$$

Experiment Design

How big a sample do I need?

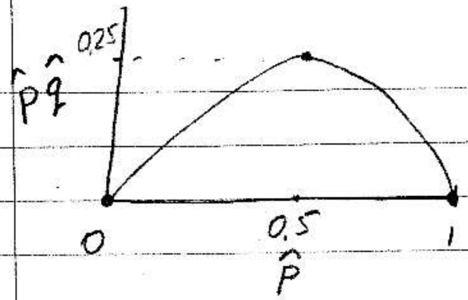
To estimate p

$$E = Z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

0.01 95% How big n?

$$\frac{E}{Z_{\alpha/2}} = \sqrt{\frac{\hat{p}\hat{q}}{n}} \qquad \frac{E^2}{Z_{\alpha/2}^2} = \frac{\hat{p}\hat{q}}{n} \qquad \frac{Z_{\alpha/2}^2}{E^2} = \frac{n}{\hat{p}\hat{q}}$$

$$n = \frac{Z_{\alpha/2}^2 \cdot \hat{p}\hat{q}}{E^2} = \frac{(1.96)^2(0.5)(0.5)}{(0.01)^2} = \frac{(1.96)^2 \cdot 0.25}{(0.01)^2} = 9604$$



When no estimate $\hat{p}$ is known

$$n = \frac{Z_{\alpha/2}^{2} \cdot 0.25}{E^{2}}$$