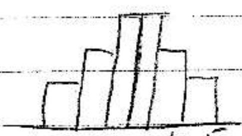
ch 6

2/27 Discrete Prob. Distribution (ch 5)
- Binomial

Continuous Prob. Distribution (Ch 6)
- Uniform
- Normal
- t distribution
- Chi square
- F distribution

$n = 5 \quad p = 0.5$



x	$P(x)$
0	
1	
2	
3	
4	
5	

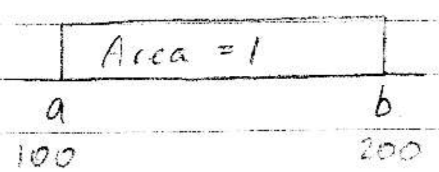
$\Sigma P(x) = 1$

weight
volume
temperature
Energy
Speed
Distance

Can't list values — 0 — 1 — infinit values

Can list values — 0, 1, 2, 3 → ∞

Uniform Distribution [a, b]



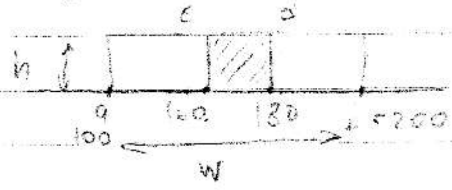
Probability will now be represented by "area under the curve"

$\frac{d-c}{b-a} = \frac{20}{100} = (d-c)\left(\frac{1}{b-a}\right)$

$h \cdot w = 1$

$w = 100$

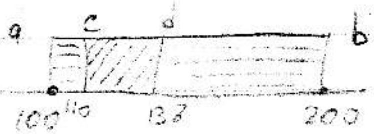
$h = \frac{1}{100}$



$d - c = 20 = w_1$

$h = \frac{1}{b-a} = \frac{1}{100} = h_1 \quad b - a = 100$

1) $x \sim U[a,b]$ $x \sim U[100, 200]$



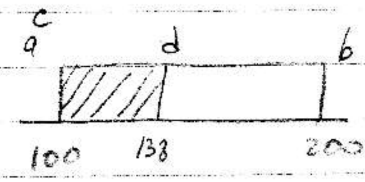
$P(110 < x < 138)$

$\neq P(x \text{ is not in } [110, 138]) =$

$= 1 - P(110 < x < 138)$

$\frac{d-c}{b-a} = \frac{138-110}{200-100} = \frac{28}{100} = 0.28$ $d-c$ part you want

2) $x \sim U[100, 200]$ $P(x < 138)$?



$P(x < 138)$

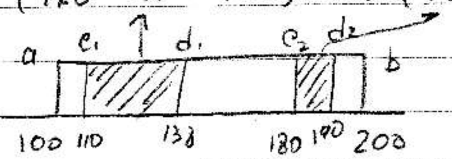
$\frac{d-c}{b-a} = \frac{138-100}{200-100} = \frac{38}{100} = 0.38$

3) $x \sim U[100, 200]$

$P(x \in \{110 \text{ to } 138\} \text{ or } x \in \{180 \text{ to } 190\})$

$P(110 < x < 138) + P(180 < x < 190) - \cancel{P(\text{Both})}$

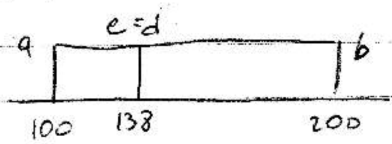
no overlap



$\frac{138-110}{200-100} + \frac{190-180}{200-100} = \frac{28}{100} + \frac{10}{100} = \frac{38}{100}$

4) $x \sim U[100, 200]$

$P(x = 138)$



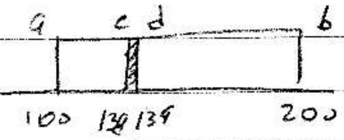
$\frac{138-138}{100} = \frac{0}{100} = 0$

138.00000000000 or 138.0000000001

any individual number $P(x = n) = 0$

5) $x \sim U[100, 200]$

$P(138 < x < 139)$



$$\frac{139-138}{200-100} = \frac{1}{100}$$

Ex. 1

100 people in a tennis tornament. 2 people - game

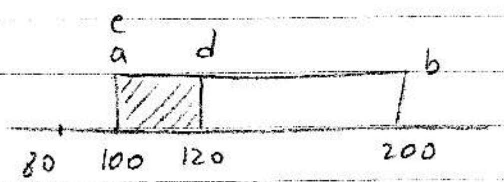
How many games will be played?

99 games Every game have a lozer

only one will be a winner

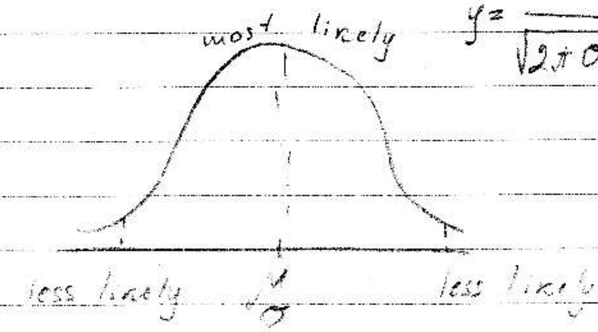
6) $x \sim U[100, 200]$

$P(80 < x < 120)$

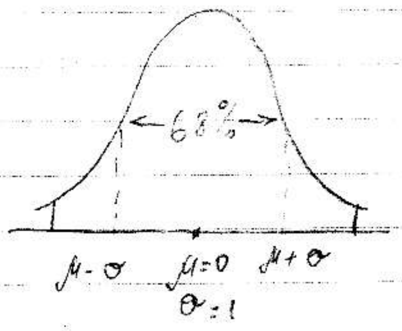


$$\frac{d-c}{b-a} = \frac{120-100}{200-100} = \frac{20}{100} = 0.2$$

Uniform density

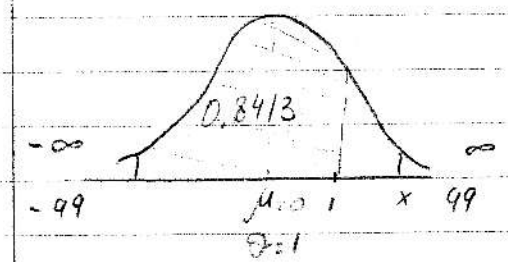


$$y = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\left\{\frac{x-\mu}{\sigma}\right\}^2}$$



p 613 Appendix A Table A-2

1.) Standart Normal distribution
has $\mu = 0$ $\sigma = 1$



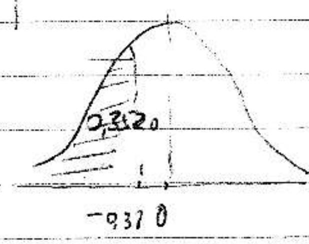
$Z = \frac{x-\mu}{\sigma} = 1$

$x = 1.38$ $Z = 1.38$

$P(Z < 1)$ $P(Z < -0.38)$

	.00	.01	.02
0.0			
0.1			
0.2			
⋮			
1.0	$\boxed{0.8413}$		

	0.8
-0.38	$\boxed{.3520}$



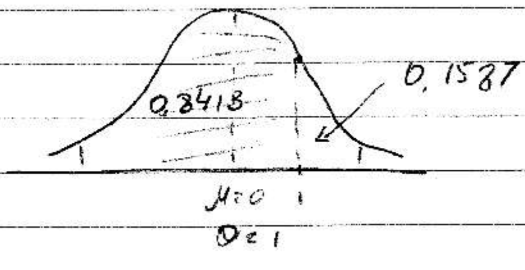
2) $P(Z < ?)$
$Z = 2.33$ $P(Z < ?)$

	0.3
2.3	- - $\boxed{.9901}$

3) $Z = -1,95$ $\quad P(Z < \;)$

$P = 0,0256$

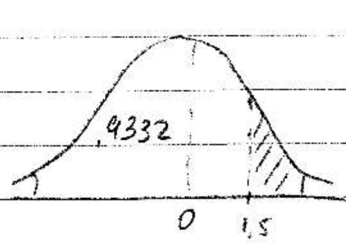
	.05
−1,9	− $\boxed{0,0256}$



$1-$

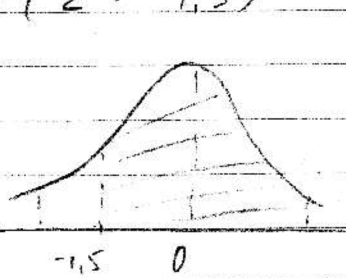
4) $x \sim U(0,1)$ $\quad P(x > 1,5)$ $\quad x = Z$

	0,00
−1,5	$\boxed{0668}$



5) $P(Z > -1,5)$ $\quad Z \cdot (-1) = Z = 1,5$

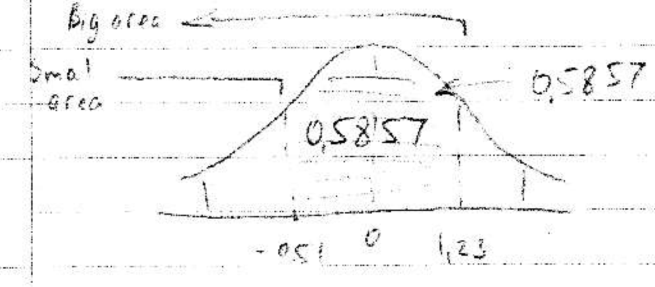
$Z = 1,5 = 0,9332$



6) $x \sim N(\mu = 0, \sigma = 1)$ varians or σ

$P(-0,51 < x < 1,23)$ $\quad Z(1,23) - Z(-0,51)$ = we look for

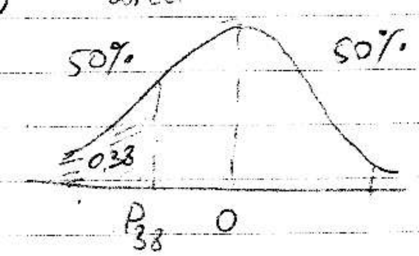
	0,03		,01
1,23	$\boxed{,8907}$	−0,5	$\boxed{,3050}$



$0,8907 - 0,3050 = 0,5857$

- you know Z and you want probability?
- you know probability and you want Z?

7)



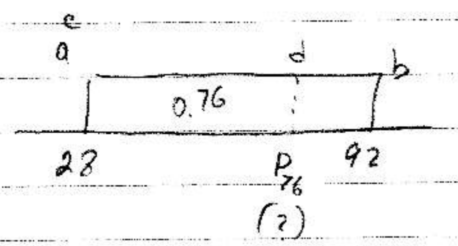
What is Z for which we find 38% of the population below Z?

	.00	.01
-0.3	- - -	$\boxed{0.3783}$

0.38

P_{38} or $Z_{38} = \boxed{-0.31}$

8) $x \sim U[28, 92]$ Find P_{76} - ?



$\frac{d - c}{b - a} = \text{prob} = 0.76$

$P_{76} = d$

$$\frac{P_{76} - 28}{92 - 28} = 0.76$$

$$\frac{P_{76} - 28}{64} = 0.76$$

$P_{76} - 28 = (0.76) \cdot 64$

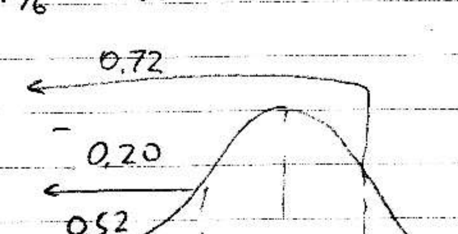
$P_{76} = 48.64 + 28$

$P_{76} = 76.64$

9) $X \sim N(0,1)$

$P(P_{20} < x < P_{72})$

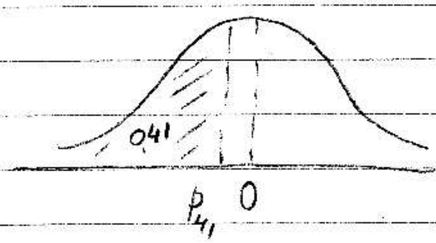
$\downarrow$ Z_1 $\quad$ $\downarrow$ Z_2



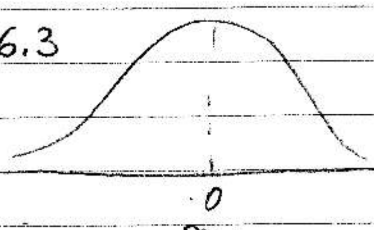
10) $X \sim N(0,1)$

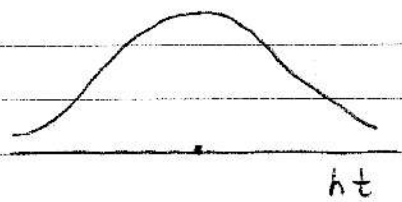
P_{41}

	0.03
	?
$-?$	$\boxed{0.41}$
-0.2	0.4090



S 6.3

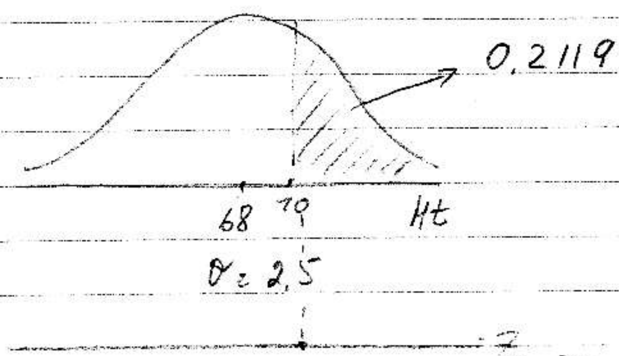




$\boxed{Z \rightarrow Prob}$ $X \rightarrow Z \rightarrow Prob$

$\boxed{Prob \rightarrow Z}$ $Prob \rightarrow Z \rightarrow X$

1) $Ht \sim N(68, 2.5)$ $P(Ht > 70)$



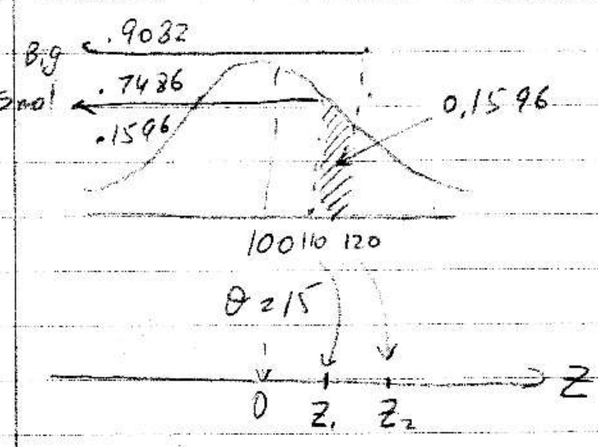
$Z = -Z$

	0.00
-0.8	$\boxed{0.2119}$

$$Z = \frac{x-\mu}{\sigma} = \frac{70-68}{2.5} = \frac{2}{2.5} = 0.8$$

p254 # 8

$x \sim N(100, 15)$



	.07
0.6	0.7486

$Z_1 = \frac{110-100}{15} = 0.67$

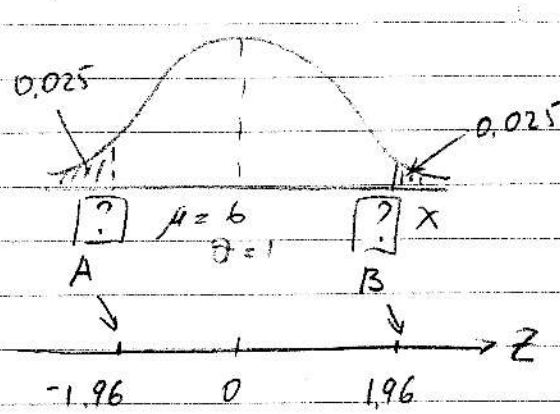
$Z_2 = \frac{120-100}{15} = 1.33$

	0.03
1.3	0.9082

$\boxed{Prob \to Z} \to x$

p256 # 23

$x \sim N(6, 1)$



	0.06
-1.9	→0.250

$\frac{A-\mu}{\theta} = -1.96$ $\frac{B-\mu}{\theta} = 1.96$

$\frac{A-6}{1} = -1.96$ $\frac{B-6}{1} = 1.96$

$A = (-1.96)\cdot 1 + 6$ $B = (1.96)\cdot 1 + 6$

$A = 4.04$ $B = 7.96$

$$\boxed{P_{2,5} = Z_{2,5} \cdot \sigma + \mu}$$

$\downarrow$

$$\boxed{P_k = Z_k \cdot \sigma + \mu}$$

Ex 1.

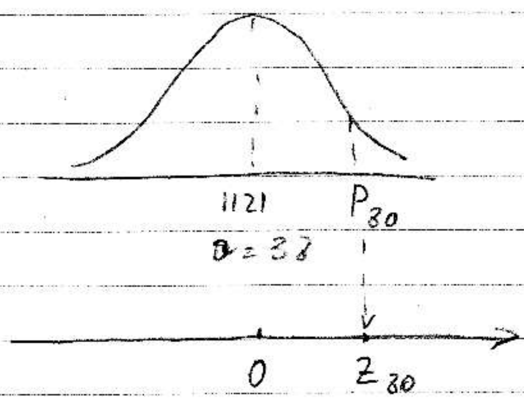
$X \sim N(1121, 38)$ $\quad P_{80}$ - ?

$P_{80} = Z_{80} \cdot 38 + 1121$

0,4

0,8 $\rightarrow$ $\boxed{0,7995}$

0,8



$P_{80} = (0,84)\,38 + 1121 =$

$= 1152,92$

Ex 2.

$\mu = 60 \quad \sigma = 12$

What is the P that a random value of X will be between 64 and 76 ?

$P(64 < x < 76)$ - ?

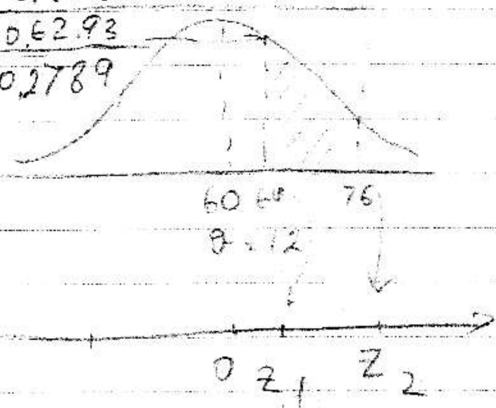
big $\leftarrow$ 0,9082

small 0,6293

0,2789

$Z_1 = \frac{64-60}{12} = 0,33$ $\quad$ 0.3 | .6293 (0.3)

$Z_2 = \frac{76-60}{12} = 1,33$ $\quad$ 1,3 | .9082 (0.03)

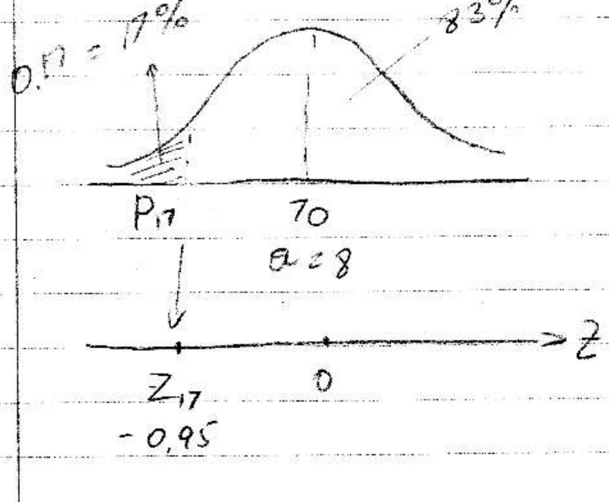


$Z = \frac{x - \mu}{\sigma}$

Ex 3

$X \sim N(\mu = 70, \sigma = 8)$

What is the 17^{th} percentile of this distribution?



P_{17} - ?

$P_{17} = Z_{17} \cdot \sigma + \mu$

$Z_{17} = 0.17$

$Z_{17} \rightarrow$ 0,5

-0,9 | $\boxed{0,17}$

0.1711

$P_{17} = (-0.95) \cdot 8 + 70 = 62.4$

S. 6-4

$X \sim U[0,1]$ Uniform distribution

consider $r = \frac{\min}{\max}$ from a sample

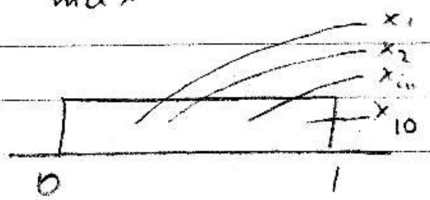
of $n = 10$ values r = ratio

PRB → RAND → Enter

0.448390514
0.898015641
min 0.197870817
0.602869484
0.219523007
max 0.99359775
0.820753107
0.614831628

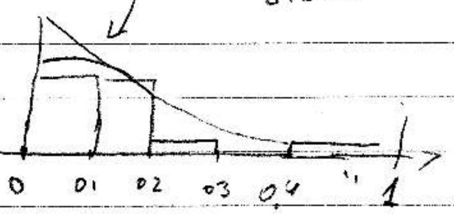
0.872760136
0.62409119

$r = \frac{min}{max} = 0.199145798$



$\frac{min}{max} = r$

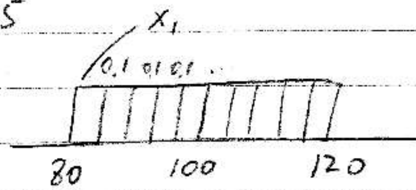
sampling distribution



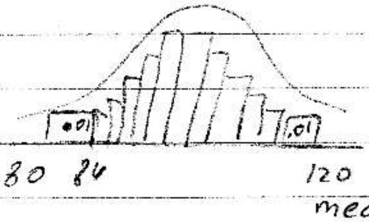
0 - 0.1	ⅠⅠⅠⅠ ⅠⅠ
0.1 - 0.2	ⅠⅠⅠⅠ ⅠⅠ
0.2 - 0.3	Ⅰ
0.3 - 0.4	
> 0.4	Ⅰ

$P(r < 0.2)$

S 6-5



$n = 2$

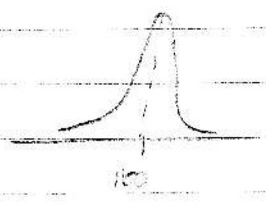


$P(80 < \bar{x}\ 84) =$

$P(x_1 < 84 \text{ and } x_2 < 84) = P(x_1 < 84)\, P(x_2 < 84) =$
$= (0.1)(0.1) = 0.01$

$n = 10$

p. 269



If x is a random variable with mean μ and σ - standart diviation then as $n \to \infty$ $\bar{x} \xrightarrow{} \sim N(\mu, \frac{\sigma}{\sqrt{n}})$

$\bar{x}$ mean