Statistics 300: Elementary Statistics Section 8-2

Hypothesis Testing

- Principles
- Vocabulary
- Problems

Principles

- Game
- I say something is true
- Then we get some data
- Then you decide whether
 - -Mr. Larsen is correct, or
 - -Mr. Larsen is a lying dog

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Risky Game

- Situation #1
- This jar has exactly (no more and no less than) 100 black marbles
- You extract a red marble
- Correct conclusion:
 - $-\mathbf{Mr.}$ Larsen is a lying dog

Principles

- My statement will lead to certain probability rules and results
- Probability I told the truth is "zero"
- No risk of false accusation

Principles

- Game
- I say something is true
- Then we get some data
- Then you decide whether
 - $-\mathbf{Mr.}$ Larsen is correct, or
 - -Mr. Larsen has inadvertently made a very understandable error

Principles

- My statement will lead to certain probability rules and results
- Some risk of false accusation
- What risk level do you accept?

Risky Game

- Situation #2
- This jar has exactly (no more and no less than) 999,999 black marbles and one red marble
- You extract a red marble
- Correct conclusion: -Mr. Larsen is mistaken

Risky Game

- Situation #2 (continued)
- Mr. Larsen is mistaken because if he is right, the one red marble was a 1-in-a-million event.
- Almost certainly, more than red marbles are in the far than just one

Risky Game

- Situation #3
- This jar has 900,000 black marbles and 100,000 red marbles
- You extract a red marble
- Correct conclusion:
 - -Mr. Larsen's statement is reasonable

Risky Game

- Situation #3 (continued)
- Mr. Larsen's statement is reasonable because it makes P(one red marble) = 10%.
- A ten percent chance is not too far fetched.

Principles (reworded)

- The statement or "hypothesis" will lead to certain probability rules and results
- Some risk of false accusation
- What risk level do you accept?

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Risky Game

- Situation #4
- This jar has 900,000 black marbles and 100,000 red marbles
- A random sample of four marbles has 3 red and 1 black
- If Mr. Larsen was correct, what is the probability of this event?

Risky Game

- Situation #4 (continued)
- Binomial: n=4, x=1, p=0.9
- Mr. Larsen's statement is <u>not</u> <u>reasonable</u> because it makes P(three red marbles) = 0.0036.
- A less than one percent chance is too far fetched.

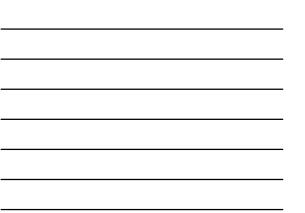
Formal Testing Method Structure and Vocabulary

- The risk you are willing to take of making a false accusation is called the <u>Significance Level</u>
- Called "alpha" or **a**
- P[Type I error]

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Two-tail	One-tail
0.20	0.10
0.10	0.05
0.05	0.025
0.02	0.01
0.01	0.005

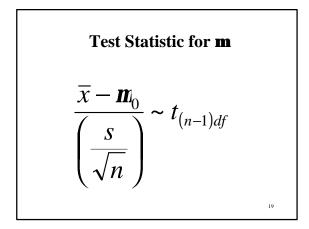


- Critical Value
 - -similar to $Z_{a/2}$ in confidence int.
 - -separates two decision regions
- Critical Region
 - -where you say I am incorrect

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Formal Testing Method Structure and Vocabulary

- Critical Value and Critical Region are based on three things:
 - $the \ hypothesis$
 - -the significance level
 - -the parameter being tested
- <u>not</u> based on data from a sample
- Watch how these work together





Test Statistic for p
(np₀>5 and nq₀>5)
$$\frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} \sim N(0,1)$$

Test Statistic for s

$$\frac{(n-1)s^{2}}{s_{0}^{2}} \sim ?^{2}_{(n-1)df}$$

- H_0 : always is = **£** or ³
- H_1 : always is 1 > or <

Formal Testing Method Structure and Vocabulary

- In the alternative hypotheses, H₁:, put the parameter on the left and the inequality symbol will point to the "tail" or "tails"
- H_1 : mp, s¹ is "two-tailed"
- H₁: **m p**, **s** < is "left-tailed"
- H_1 : **m** p, **s** > is "right-tailed"

Formal Testing Method Structure and Vocabulary

- Example of Two-tailed Test
 - $-\mathbf{H}_0: \mathbf{m} = 100$
 - $-\mathbf{H_{1}:\,m^{1}\,\,100}$

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- Example of Two-tailed Test
 - $-\mathbf{H}_0: \mathbf{m} = \mathbf{100}$
 - $-H_1\textbf{: }m^1 \ 100$
- Significance level, **a** = 0.05
- Parameter of interest is **m**

Formal Testing Method Structure and Vocabulary

- Example of Two-tailed Test
 - $-\mathbf{H}_0: \mathbf{m} = 100$
 - $-\mathbf{H_{1}:m^{1}}\ \mathbf{100}$
- Significance level, **a** = 0.10
- Parameter of interest is **m**

Formal Testing Method Structure and Vocabulary

- Example of Left-tailed Test
 - –H₀: р ^з 0.35
 - $-H_1: p < 0.35$

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- Example of Left-tailed Test
 - -H₀: р ^з 0.35 -H₁: р < 0.35
- Significance level, **a** = 0.05
- Parameter of interest is "p"

Formal Testing Method Structure and Vocabulary

- Example of Left-tailed Test
 - –H₀: р ^з 0.35
 - $-H_1: p < 0.35$
- Significance level, **a** = 0.10
- Parameter of interest is "p"

Formal Testing Method Structure and Vocabulary

- Example of Right-tailed Test
 - $-H_0: s \ \pounds \ 10$
 - $-H_1: s > 10$

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- Example of Right-tailed Test -H₀: s £ 10
 - $-H_1: s > 10$
- Significance level, **a** = 0.05
- Parameter of interest is **s**

Formal Testing Method Structure and Vocabulary 31

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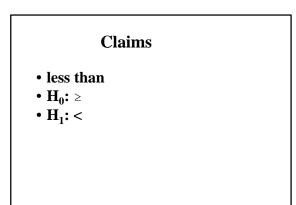
- Example of Right-tailed Test
 - $-\mathbf{H_0: s £10}$
 - $-H_1: s > 10$
- Significance level, **a** = 0.10
- Parameter of interest is **s**

Claims

• is, is equal to, equals	=
• less than	<
 greater than 	>
 not, no less than 	\geq
 not, no more than 	\leq
• at least	\geq
• at most	\leq
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Claims

- is, is equal to, equals
- H₀: = H₁: ¹



Claims

- greater than H_0 : \leq
- H₁: >

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Claims

- not, no less than
- H_0 : \geq
- H₁: <

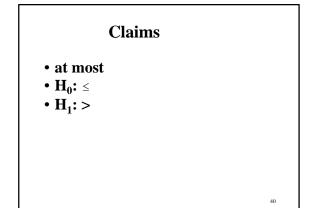
Claims

- not, no more than
- H_0 : \leq
- H₁: >

Claims

- at least
- H_0 : \geq
- H₁: <

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Structure and Vocabulary

- Type I error: Deciding that H₀: is wrong when (in fact) it is correct
- Type II error: Deciding that H₀: is correct when (in fact) is is wrong

Structure and Vocabulary

- Interpreting the test result
 - The hypothesis is not reasonable
 - $\mbox{The Hypothesis is reasonable}$
- Best to define reasonable and unreasonable before the experiment so all parties agree

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Traditional Approach to Hypothesis Testing

Test Statistic

- Based on Data from a Sample and on the Null Hypothesis, H₀:
- For each parameter (**m p**, **s**), the test statistic will be different
- Each test statistic follows a probability distribution

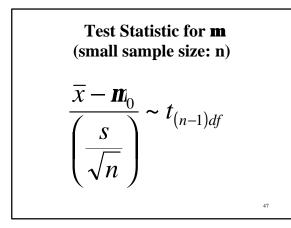
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Traditional Approach

- Identify parameter and claim
- Set up H₀: and H₁:
- Select significance Level, **a**
- Identify test statistic & distribution
- Determine critical value and region
- Calculate test statistic
- Decide: "Reject" or "Do not reject"

Next three slides are repeats of slides 19-21



Test Statistic for p
(np₀>5 and nq₀>5)
$$\frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} \sim N(0,1)$$

