Statistics 300: Elementary Statistics

Section 8-2

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Hypothesis Testing

- Principles
- Vocabulary
- Problems

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Principles

- Game
- I say something is true
- Then we get some data
- Then you decide whether
  - Mr. Larsen is correct, or
  - Mr. Larsen is a lying dog

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Risky Game

- Situation #1
- This jar has exactly (no more and no less than) 100 black marbles
- You extract a red marble
- Correct conclusion:
  - Mr. Larsen is a lying dog

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Principles

- My statement will lead to certain probability rules and results
- Probability I told the truth is “zero”
- No risk of false accusation

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Principles

- Game
- I say something is true
- Then we get some data
- Then you decide whether
  - Mr. Larsen is correct, or
  - Mr. Larsen has inadvertently made a very understandable error

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Principles

- My statement will lead to certain probability rules and results
- Some risk of false accusation
- What risk level do you accept?

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Risky Game

- Situation #2
- This jar has exactly (no more and no less than) 999,999 black marbles and one red marble
- You extract a red marble
- Correct conclusion:
  - Mr. Larsen is mistaken

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Risky Game

- Situation #2 (continued)
- Mr. Larsen is mistaken because if he is right, the one red marble was a 1-in-a-million event.
- Almost certainly, more than red marbles are in the far than just one

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Risky Game

- Situation #3
- This jar has 900,000 black marbles and 100,000 red marbles
- You extract a red marble
- Correct conclusion:
  - Mr. Larsen’s statement is reasonable

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Risky Game

- Situation #3 (continued)
- Mr. Larsen’s statement is reasonable because it makes P(one red marble) = 10%.
- A ten percent chance is not too far fetched.

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Principles (reworded)

- The statement or “hypothesis” will lead to certain probability rules and results
- Some risk of false accusation
- What risk level do you accept?

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Risky Game

- **Situation #4**
- **This jar has 900,000 black marbles and 100,000 red marbles**
- **A random sample of four marbles has 3 red and 1 black**
- **If Mr. Larsen was correct, what is the probability of this event?**

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Risky Game

- **Situation #4 (continued)**
- **Binomial: n=4, x=1, p=0.9**
- **Mr. Larsen's statement is <u>not reasonable</u> because it makes P(three red marbles) = 0.0036.**
- **A less than one percent chance is too far fetched.**

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Formal Testing Method Structure and Vocabulary

- **The risk you are willing to take of making a false accusation is called the <u>Significance Level</u>**
- **Called "alpha" or** a
- **P[Type I error]**

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Conventional a levels

Two-tail	One-tail
0.20	0.10
0.10	0.05
0.05	0.025
0.02	0.01
0.01	0.005

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Formal Testing Method Structure and Vocabulary

- **Critical Value**
  - similar to $Z_{a/2}$ in confidence int.
  - separates two decision regions
- **Critical Region**
  - where you say I am incorrect

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Formal Testing Method Structure and Vocabulary

- **Critical Value and Critical Region are based on three things:**
  - the hypothesis
  - the significance level
  - the parameter being tested
- **<u>not</u> based on data from a sample**
- **Watch how these work together**

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Test Statistic for m

$$\frac{\bar{x}-m_0}{\left(\frac{s}{\sqrt{n}}\right)} \sim t_{(n-1)df}$$

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Test Statistic for p (np$_0$>5 and nq$_0$>5)

$$\frac{\hat{p}-p_0}{\sqrt{\frac{p_0 q_0}{n}}} \sim N(0,1)$$

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Test Statistic for s

$$\frac{(n-1)s^2}{s_0^2} \sim ?^2_{(n-1)df}$$

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Formal Testing Method Structure and Vocabulary

- H_0: always is $=$ $\leq$ or $\geq$
- H_1: always is $\neq$ $>$ or $<$

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Formal Testing Method Structure and Vocabulary

- In the alternative hypotheses, H_1:, put the parameter on the left and the inequality symbol will point to the “tail” or “tails”
- H_1: μ, p, σ $\neq$ is “two-tailed”
- H_1: μ, p, σ $<$ is “left-tailed”
- H_1: μ, p, σ $>$ is “right-tailed”

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Formal Testing Method Structure and Vocabulary

- Example of Two-tailed Test
  - H_0: $\mu = 100$
  - H_1: $\mu \neq 100$

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Formal Testing Method Structure and Vocabulary

- **Example of Two-tailed Test**
  - **H_0: $\mu = 100$**
  - **H_1: $\mu \neq 100$**
- **Significance level, $\alpha = 0.05$**
- **Parameter of interest is μ**

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Formal Testing Method Structure and Vocabulary

- **Example of Two-tailed Test**
  - **H_0: $\mu = 100$**
  - **H_1: $\mu \neq 100$**
- **Significance level, $\alpha = 0.10$**
- **Parameter of interest is μ**

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Formal Testing Method Structure and Vocabulary

- **Example of Left-tailed Test**
  - **H_0: $p \geq 0.35$**
  - **H_1: $p < 0.35$**

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Formal Testing Method Structure and Vocabulary

- **Example of Left-tailed Test**
  - $H_0: p \geq 0.35$
  - $H_1: p < 0.35$
- **Significance level,** $\alpha = 0.05$
- **Parameter of interest is "p"**

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Formal Testing Method Structure and Vocabulary

- **Example of Left-tailed Test**
  - $H_0: p \geq 0.35$
  - $H_1: p < 0.35$
- **Significance level,** $\alpha = 0.10$
- **Parameter of interest is "p"**

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Formal Testing Method Structure and Vocabulary

- **Example of Right-tailed Test**
  - $H_0: \sigma \leq 10$
  - $H_1: \sigma > 10$

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Formal Testing Method Structure and Vocabulary

- **Example of Right-tailed Test**
  - **H_0:** s £ **10**
  - **H_1:** s **> 10**
- **Significance level,** a **= 0.05**
- **Parameter of interest is** s

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Formal Testing Method Structure and Vocabulary

- **Example of Right-tailed Test**
  - **H_0:** s £ **10**
  - **H_1:** s **> 10**
- **Significance level,** a **= 0.10**
- **Parameter of interest is** s

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Claims

• **is, is equal to, equals**	=
• **less than**	<
• **greater than**	>
• **not, no less than**	≥
• **not, no more than**	≤
• **at least**	≥
• **at most**	≤

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Claims

- is, is equal to, equals
- H_0: =
- H_1: 1

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Claims

- less than
- H_0: $\geq$
- H_1: <

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Claims

- greater than
- H_0: $\leq$
- H_1: >

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Claims

- not, no less than
- H_0: $\geq$
- H_1: $<$

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Claims

- not, no more than
- H_0: $\leq$
- H_1: $>$

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Claims

- at least
- H_0: $\geq$
- H_1: $<$

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Claims

- at most
- H_0: $\leq$
- H_1: >

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Structure and Vocabulary

- Type I error: Deciding that H_0: is wrong when (in fact) it is correct
- Type II error: Deciding that H_0: is correct when (in fact) is is wrong

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Structure and Vocabulary

- Interpreting the test result
  - The hypothesis is not reasonable
  - The Hypothesis is reasonable
- Best to define reasonable and unreasonable before the experiment so all parties agree

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Traditional Approach to Hypothesis Testing

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Test Statistic

- Based on Data from a Sample and on the Null Hypothesis, H_0:
- For each parameter (m, p, s), the test statistic will be different
- Each test statistic follows a probability distribution

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Traditional Approach

- Identify parameter and claim
- Set up H_0: and H_1:
- Select significance Level, a
- Identify test statistic & distribution
- Determine critical value and region
- Calculate test statistic
- Decide: “Reject” or “Do not reject”

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Next three slides are repeats of slides 19-21

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Test Statistic for m (small sample size: n)

$$\frac{\bar{x} - m_0}{\left(\frac{s}{\sqrt{n}}\right)} \sim t_{(n-1)df}$$

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Test Statistic for p (np_0>5 and nq_0>5)

$$\frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} \sim N(0,1)$$

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Test Statistic for s

$$\frac{(n-1)s^2}{s_0^{\,2}} \sim ?^2_{(n-1)df}$$

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