Statistics 1:
Elementary Statistics

Section 4-4

Section 4-5

Probability

- **Chapter 3**
  - –**Section 2: Fundamentals**
  - –**Section 3: Addition Rule**
  - –**Section 4: Multiplication Rule #1**
  - –**Section 5: Multiplication Rule #2**
  - –**Section 6: Simulating Probabilities**
  - –**Section 7: Counting**

Multiplication Rule #1

- **P(A <u>and</u> B) = ?**
  - –**Two rolls: P(2 <u>and then</u> 5) = ?**
  - –**Two dice:**
    P(sum<9 <u>and</u> both odd) = ?

P(A and B)

- Two rolls:
- A : first die is 2
- B : second die is 5
- P(A and B) = ?

Value of Die #1

Value of Die #2	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

P(A and B)

- Circled event on last slide
- A : first die is 2
- B : second die is 5
- P(A and B) = 1 / 36

P(A **and** B)

- Circled event on last slide
- A : first die is 2
- B : second die is 5

$$P(A) \cdot P(B) = \left(\frac{1}{6}\right) \cdot \left(\frac{1}{6}\right) = \frac{1}{36}$$

P(A **and** B)

- Does this always work?

$$P(A \text{ and } B) = P(A) \cdot P(B)\ ?$$

- Of course not – try the next problem using the two dice table.

P(A **and** B)

- Two dice:
- A = sum < 9
- B = both are odd
- P(A) =

26 events where sum is < 9.

Value of Die #2	Value of Die #1: 1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

9 events where both are odd.

Value of Die #2	Value of Die #1: 1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
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But only 8 of these 9 events have sum < 9

In this case, it is clear that the answer must be $\frac{8}{36} = 0.222$ **which is not equal to**

$$P(A) \cdot P(B) = \left[\frac{26}{36}\right]\left[\frac{9}{36}\right] = 0.181$$

To save the situation, we must use the formal multiplication rule :

$$P(A \text{ and } B) = P(A) \times P(B \mid A)$$

$$= \frac{26}{36} \times \frac{8}{26} = \frac{8}{36} = 0.222$$

Conditional Probability

$$P(A \mid B)$$

"probability of A given B" that is, B has happened or must happen

Start with the Multiplication Rule

$$P(A \text{ and } B) = P(A) \cdot P(B \mid A)$$

or

$$P(A \text{ and } B) = P(B) \cdot P(A \mid B)$$

And rearrange it.

The Multiplication Rule rearranged

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$$

or

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

Concept of "Independent" outcomes

If event A does not alter the <u>probability</u> of event B, and vice versa, then A and B are "independent" and

$$P(A|B) = P(A)$$

$$P(B|A) = P(B)$$