Statistics 1: Introduction to Probability and Statistics

Section 3-3

Three Statistics Describing Variation

- **Range**
- **Standard Deviation**
- **Variance**

Range

- **Not the Midrange (measure of the center)**
- **Difference between the largest and the smallest values**
- **Max - Min**
- **Uses only these two values**

Standard Deviation

- **Uses all of the data**
- **The characteristic measure of variability**

$$s = \sqrt{\frac{\sum (x - \bar{x})^2}{n-1}}$$

Variance

- **Square of the standard deviation**

$$s^2 = \frac{\sum (x - \bar{x})^2}{n-1}$$

Mean and Variance

- **Sum of squares is numerator in variance formula**

$$s^2 = \frac{\sum (x - \bar{x})^2}{n-1}$$

- **Mean makes** $\sum (x - \bar{x})^2$ **as small as it can be**

Mean and Variance

- **Mean makes the sum of squares as small as it can be**

$$\sum (x - \bar{x})^2$$

- **A value other than $\bar{x}$ makes the sum of squares bigger**

Meaning of the Standard Deviation

- **What can you learn from the**
- **Range Rule?**
- **Empirical Rule?**
- **Chebyshev's Theorem?**

Range Rule

- **The standard deviation is approximately equal to the range divided by 4.**
- **Or, range is about (4)(s)**
- **"Quick and Dirty" estimate**

Range rule

$$s \cong \frac{Range}{4} = \frac{(\max - \min)}{4}$$

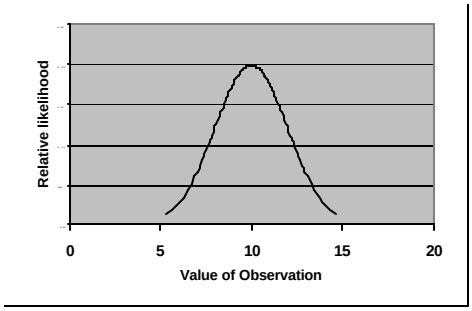
Range rule

$$Range \cong (s)(4)$$

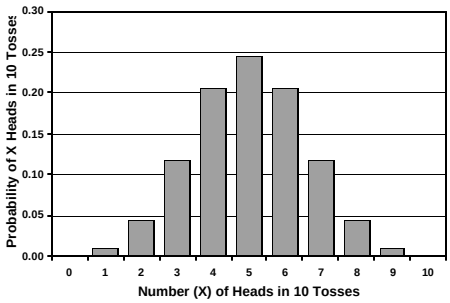
Range rule

$$\frac{Range}{s} = 4$$

Bell-Shaped Distribution



Bell-Shaped Distribution



Empirical Rule

- **For "bell-shaped" distributions, approximately:**
- **68% of values within** μ **+/- 1**σ
- **95% of values within** μ **+/- 2**σ
- **99.7% of values within** μ **+/- 3**σ

Chebyshev's Theorem

- **For any distribution and for k = 2 or more,**
- **The smallest possible percentage of the values that can lie within μ +/- kσ is $(1 - 1/k^2)$**

Standard Deviation Problems

- **Given :**
  - **the mean is 100**
  - **the standard deviation is 6**
  - **bell-shaped distribution**
- **Estimate the proportion of data that lies between 88 and 112.**

Standard Deviation Problems

- **Given :**
  - **the mean is 100**
  - **the standard deviation is 6**
- **What is the smallest percentage of values that could possibly lie between 82 and 118?**

Standard Deviation Problems

- **Given that the standard deviation is equal to 64, estimate the difference between the largest and the smallest value.**

Standard Deviation Problems

- **In a random sample of 40 values, the smallest value was 30, the largest value was 430, and the standard deviation was 20. Do the results seem reasonable?**