Statistics 300 Name: Solution Summer 2011
Instructor: Lawrence C. Larsen MtuWTh 5:30 – 8:15 pm

Exam #2

(8 points; 8 minutes)

1. **The "ABC" company has a web page on which customers can give them feedback about ABC's services. Last year, the web page collected 776 responses, 494 of which were from dissatisfied customers who were unhappy with ABC's services. Use these data to construct a 92% confidence interval for the proportion of ABC customers that are happy with ABC's services.**

$N = 776$

unhappy $= 494$

happy $= 282$

$\hat{p} = \frac{282}{776} = 0.363$

$\hat{q} = 0.637$

confidence $= 0.92$

$\alpha = 1 - 0.92 = 0.08$

$\alpha/2 = 0.04$

$Z_{\alpha/2} = 1.75$

$$92\% \, CI(p) = \hat{p} \pm Z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$= 0.363 \pm 1.75\sqrt{\frac{(.363)(.637)}{776}}$$

$$= 0.363 \pm 0.030$$

$$[0.333 < p < 0.393]$$

	.05
-1.7	$\boxed{.0401}$

Considering your confidence interval and the data on which it is based, is it reasonable for the customer service manager to claim that more than 90% of ABC's customers are happy with ABC's services? (Circle your answer and explain why.)

YES **NO** **Why?** Because the sample was a convenience sample of self-selected "volunteers." Such samples are known to be biased, so the CI is not reliable.

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2. **The "DEF" company makes "curly-fry" light bulbs (also called compact flourescent bulbs). DEF's light bulbs are supposed to weigh 244 grams on average. Use the data in the box for a random sample of seven (7) of DEF's light bulbs to make a 95% confidence interval for the mean weight of all the curly-fry light bulbs manufactured by the DEF company.** (The weights of individual bulbs have a bell-shaped distribution.)

Data:
244
252
247
249
241
241
243

$\alpha = 1 - \text{confidence}$
$= 1 - 0.95$
$= 0.05$ in 2 tails

$\alpha/2 = 0.025$ in 1 tail

$t_{\alpha/2} = 2.447$

$n = 7$
$\bar{x} = 245.3$
$s = 4.19$
d.f. $= 6$

$$95\% \ CI(\mu) = \bar{x} \pm t_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right)$$
$$= 245.3 \pm 2.447\left(\frac{4.19}{\sqrt{7}}\right)$$
$$= 245.3 \pm 3.88$$

$$[241.4 < \mu < 249.2]$$

Based on your confidence interval, is it reasonable to claim that DEF's curly-fry light bulbs weigh on average what they are supposed to weigh?

YES **NO** **Why?** μ is supposed to be 244, and 244 is in the CI, the "reasonable range for the truth."

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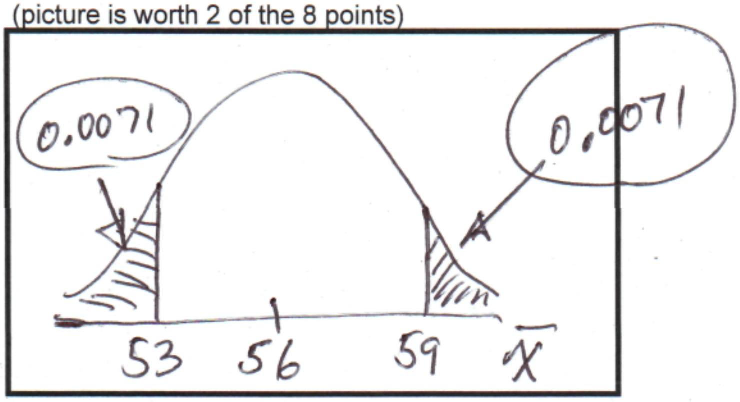
3. **The "GHI" company makes small gasoline-powered cars. The fuel efficiency (miles per gallon) of GHI's cars is normally distributed with an average of 56 miles per gallon and a standard deviation of 3 miles per gallon. What is the probability that a random sample of six (6) of GHI's cars will have an average fuel efficiency less than 53 miles per gallon or (yes, "or") greater than 59 miles per gallon?**

(picture is worth 2 of the 8 points)

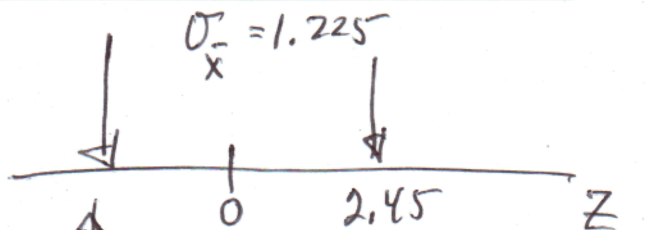


$X \sim N(\mu = 56, \sigma = 3)$

$\bar{X} \sim N(\mu_{\bar{x}} = 56, \sigma_{\bar{x}} = 1.225)$

for $n = 6$

$\sigma_{\bar{x}} = \sigma/\sqrt{n} = 3/\sqrt{6} = 1.225$



Prob to the right of 2.45 = Prob to the left of −2.45.

$\boxed{2.45} = \dfrac{59-56}{1.225}$

	.05
−2.4	$\boxed{0.0071}$

$\dfrac{53-56}{1.225} = \boxed{-2.45}$

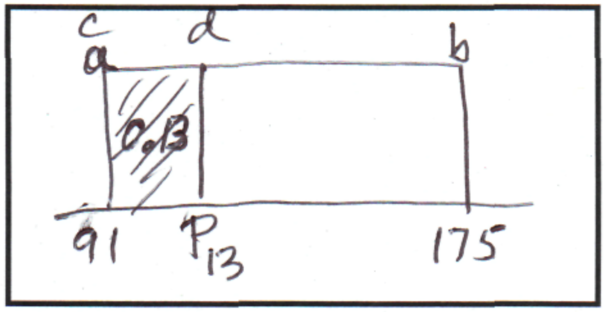
$P(\bar{X} < 53 \ \underline{\text{or}} \ \bar{X} > 59)$

$= 0.0071 + 0.0071$

Answer → $= 0.0142$

(5 points; 5 minutes)

4. **A population follows the uniform distribution on the interval [91, 175]. What is the 13th percentile of this population? The picture is worth 2 of the five points.**



$$\frac{d-c}{b-a} = 0.13 = \frac{P_{13} - 91}{175 - 91}$$

$$0.13 = \frac{P_{13} - 91}{84}$$

$$(0.13)(84) = P_{13} - 91$$

$$(0.13)(84) + 91 = P_{13}$$

$$101.92 = P_{13}$$

(6 points; 6 minutes)

5. **The "JKL" company will study a city's garbage dump to see if the "stuff" it contains is valuable enough to be worth "mining". If the value of the stuff is at least $10 per ton, JKL will start mining the dump. JKL will take samples from the dump and determine the value ($ per ton) of each one. JKL wants to be 99% certain that the average value of their samples will be within $0.50 of the true average value in the whole dump. The first 20 samples have an average value of $9.60 with standard deviation of $3.20. For a successful study, how many total samples should JKL plan to take from the dump?**

sample size needed to estimate μ

$$n = \left[\frac{Z_{\alpha/2} \cdot \hat{\sigma}}{E}\right]^2 = \left[\frac{(2.575)(3.20)}{0.5}\right]^2$$

$$= 271.59 \uparrow 272$$

confidence = 99%

$\alpha = 0.01$

$\alpha/2 = 0.005$

$Z_{\alpha/2} = 2.575$

$\hat{\sigma} = \$3.20 \quad E = \$0.50$

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(8 points; 8 minutes)

6. **The "MNO" company provides health care services to 590,000 Medicare patients. The annual cost for each of their patients (590,00 values) has been Normally distributed with a mean of \$9,600 and a standard deviation of \$4,200. A random sample of 30 patient costs this year has a mean of \$9,900 and a standard deviation of \$4,500. MNO expected the increase in average cost, but MNO is worried about the possibility that the variation in costs has increased. Use their sample results to test the Medicare Administrator's claim that the standard deviation of cost per-patient has not increased by more than \$200.**

   (Use a 5% significance level for this test.)

claim: $\sigma \le 4200 + 200$

$\sigma \le 4400$

$\boxed{n = 30 \quad d.f. = 29 \quad S = 4500}$

H_0: $\sigma \le 4400$

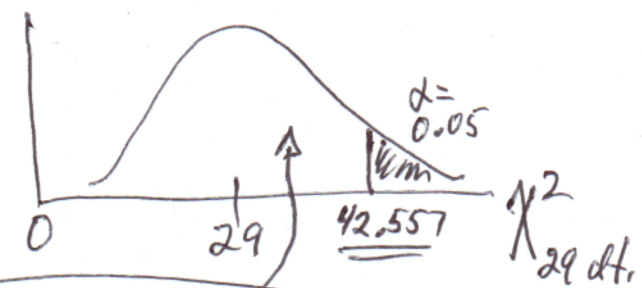
H_1: $\sigma > 4400$

$\alpha = 0.05$ right tail

Test Statistic

$$\frac{(n-1)s^2}{\sigma_0^2} = \frac{(29)(4500)^2}{(4400)^2} = 30.33$$

critical region



Do not reject Ho:

(6 points; 6 minutes)

7. **The "PQR" company sells flood insurance in Mississippi. PQR wants to start selling flood insurance in Alabama (the state on the east side of Mississippi). PQR will carry out a survey in Alabama to estimate the percentage of the people in Alabama that would consider buying flood insurance. PQR wants to have 90% confidence that the difference between their survey percentage and the true percentage for all people in Alabama will not exceed 3% points. Last year, PQR did a study like this in Mississippi and found that about 25% of the people would buy flood insurance. The reach their goals, how many Alabama residents should PQR select (at random) to survey?**

sample size to estimate p.

$$n = \frac{(z_{\alpha/2})^2 \hat{p}\hat{q}}{(E)^2} = \frac{(1.645)^2(.25)(.75)}{(.03)^2}$$

$= 563.8 \uparrow 564$

confidence $= 0.90$

$\alpha = 0.10$

$\alpha/2 = 0.05$

$z_{\alpha/2} = 1.645$

$\hat{p} = 0.25$

$\hat{q} = 0.75$

$E = 0.03$

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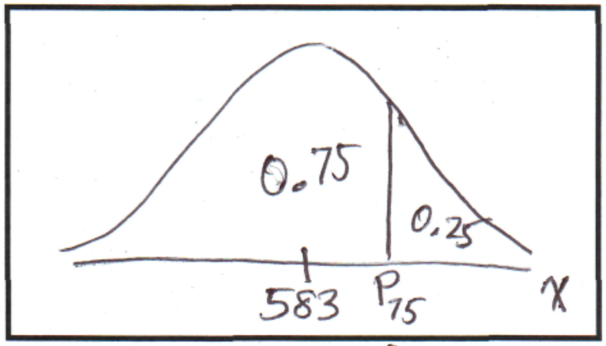
(5 points; 4 minutes)

8. For the Normal distribution with mean = 583 and standard deviation = 56, what is the value that separates the lower 75% of the distribution from the upper 25% ?
The picture is required and is worth 2 points.



$\sigma = 56$

$$\frac{P_{75} - \mu}{\sigma} = Z_{75}$$

$$\frac{P_{75} - 583}{56} = 0.67$$

$$P_{75} = (0.67)(56) + 583$$
$$= 620.5$$

$0.67 = Z_{75}$ $\quad Z$

Z	.07
0.6	$\boxed{.7486}$

(9 points; 10 minutes)

9. The "STU" company runs a private "weight loss" clinic. Last year, their customers lost an average of 22 pounds. The data below are for a random sample of 5 customers this year. Test the manager's claim that customers this year will lose less weight on average compared to last year. The distribution of weight loss for all customers is bell-shaped.
(Let α be 0.025 for this test.) claim: $\mu_{Now} < \mu_{last\ year}$

$\mu < 22$

Client	Weight Loss
1	14
2	15
3	23
4	14
5	13

* weights in pounds

H_0: $\mu \geq 22$

H_1: $\mu < 22$

$\alpha = 0.025$ left tail

$n = 5$
$df = 4$
$\bar{x} = 15.8$
$s = 4.09$

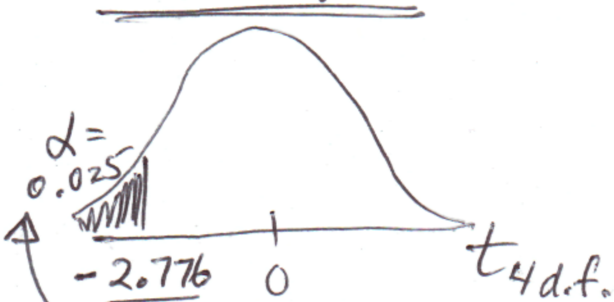
test statistic

$$\frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

$$= \frac{15.8 - 22}{4.09/\sqrt{5}}$$

$$= -6.2/1.829 = -3.390$$

critical region



Reject H_0:

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(8 points; 9 minutes)

10. The "VWX" company provides engineering services to its customers. VWX has been charged with racial discrimination in hiring its engineers. The company hired 62 engineers in the last 5 years, 55 were pale and 7 were not pale. The whole population of engineers available to VWX for hiring was 73% pale and 27% not pale. Use these data (as if only these data are relevant) **to test the claim against VWX that they are biased and tend to hire a greater proportion of pale engineers than the proportion in the population available to VWX.** (Let P[Type I Error] = 0.05 for this test.)

$N = 62$

$\hat{p} = 55/62 = 0.8871$

$\hat{q} = 0.1129$

claim: $p > 0.73$

H_0: $p \le 0.73$

H_1: $p > 0.73$

$\alpha = 0.05$ right tail

critical region

Reject Ho:

Test Statistic

$$\frac{\hat{p} - p_0}{\sqrt{\dfrac{p_0 q_0}{n}}} = \frac{0.8871 - 0.73}{\sqrt{\dfrac{(0.73)(0.27)}{62}}} = \frac{0.1571}{0.0564} = \boxed{2.785}$$

Note: An analysis such as this is not enough to convict a company of discrimination in hiring. A review of every hiring decision may be needed. The case-by-case evidence would have to show that preference was given to pale applicants who were less qualified than their non-pale competitors, or that pale applicants were preferred disproprtionately when competitors were equally qualified.

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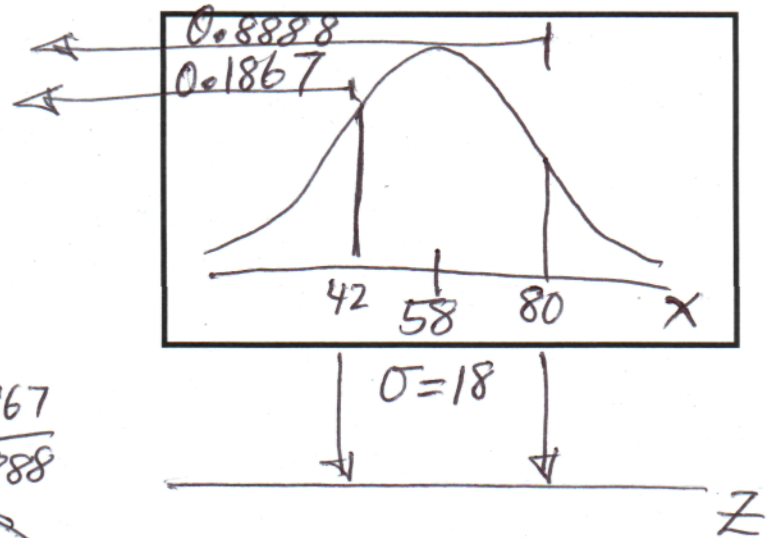
(7 points; 7 minutes)

11. The random variable X is Normally distributed, where $\mu = 58$ and $\sigma = 18$. What is the probability that a random X will be less than 42 given that X must be less than 80 ? (The picture is worth 2 of the 7 points.)

$$Z(42) = \frac{42-58}{18} = -0.89$$

$$Z(80) = \frac{80-58}{18} = 1.22$$



$$P(x < 42 \mid x < 80)$$

$$= \frac{P(x<42 \text{ and } x<80)}{P(x<80)} = \frac{0.1867}{0.8888}$$

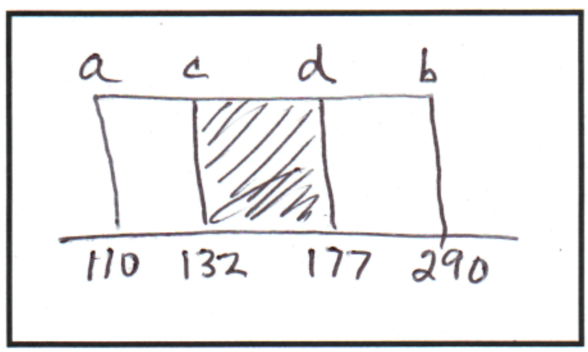
$$= 0.21$$

12. (4 points; 3 minutes)
For parts "a" and "b", use the fact that X ~ U[110, 290].

(a) What is the probability that a random value of X will be greater than 132 and less than 177 ? (The picture is required and is worth 2 points.)

$$\text{Prob} = \frac{d-c}{b-a} = \frac{177-132}{290-110}$$

$$= \frac{45}{180} = 0.25$$



(2 points; 3 minutes)
(b) What is the probability that two random values will both be between 132 and 177 ?

$$P(132 < x_1 < 177) = 0.25$$

$$P(132 < x_2 < 177) = 0.25$$

$$P(\text{both}) = (0.25)(0.25) = 0.0625$$