

Math 372
Fall 2025
Exam 3, December 4

No books, notes, scratch paper, phones.
Please show all your work and clearly mark your answers.
Problems are 10 points unless noted.
If a problem is too hard, move on to an easier one.
No calculators on problems 1-8.

Page	Pts	Possible
1		5
2		10
3		25
4		25
5		20
6		15
Total		100

Name (printed):

Key

Name (signature):

Score for the
class so far:

_____ out of _____ points

Percent:

_____ %

Approximate letter grade:

To earn a grade of _____ I would need about
of the points in the rest of the class.

1) Use mathematical induction to prove that $2^n > 50n$ for all $n \geq 9$.

(5 pts)

If $n = 9$,

$$\text{LHS} = 2^9 = 512$$

$$\text{RHS} = 450. \text{ Yes, valid for } n = 9.$$

Assume $2^n > 50n$

times 2

$$2^{n+1} > 50n \cdot 2$$

$$2^{n+1} > 100n$$

$$2^{n+1} > 50n + 50n$$

Since $n \geq 1$, $50n \geq 50$

$$2^{n+1} > 50n + 50$$

$$2^{n+1} > 50(n+1)$$

The statement is valid for $n+1$.

Thus, $2^n > 50n$ for all $n \geq 9$.

2) Solve:

$$\begin{cases} x+y+z=4 \\ 2x-y+3z=25 \\ 5x+2y+z=12 \end{cases} \rightarrow \text{times } 2$$

$$2x+2y+2z=8 \quad \uparrow$$

$$5x+2y+z=12$$

$$3x+4z=29$$

$$\downarrow \quad 3x-z=4$$

$$5z=25$$

$$\begin{aligned} z &= 5 \\ x &= 3 \\ y &= -4 \end{aligned}$$

$$3-4+5=4 \quad \checkmark$$

$$6+4+15=25 \quad \checkmark$$

$$15-8+5=12 \quad \checkmark$$

3) For each sequence, write "arithmetic sequence", "geometric sequence", or "neither". Justify your answers.

a) 25, 36, 49, 64, ...

neither

Differences are 11, 13, 15

(5 pts)
Ratios are $\frac{36}{25}$, $\frac{49}{36}$, $\frac{64}{49}$

b) 19, 27, 35, 43, ...

arithmetic

common difference is 8

c) 7, -14, 28, -56, ...

geometric

common difference is -2

4) Find these logs, if possible:

a) $\log_{10}(1,000,000,000)$

9

b) $\log_2(8\sqrt{2})$

$3\frac{1}{2}$

c) $\ln\left(\frac{1}{e^4}\right)$

-4

d) $\log_4(-4)$

undefined

5) Find the sum of this infinite series:

$$8 + 2 + \frac{1}{2} + \dots$$

$$\text{ratio} = \frac{1}{4}$$

$$S = \frac{8}{1 - \frac{1}{4}} = \frac{8}{\frac{3}{4}} = \boxed{\frac{32}{3}}$$

6) Find the sum of the first 100 terms of this series:

$$-32 + (-26) + (-20) + \dots$$

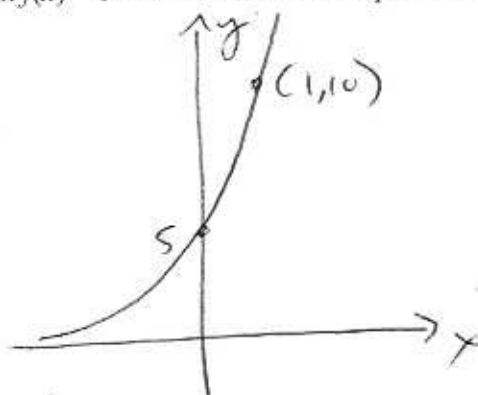
common difference is 6

$$a_{100} = -32 + 6 \cdot 99 = 562$$

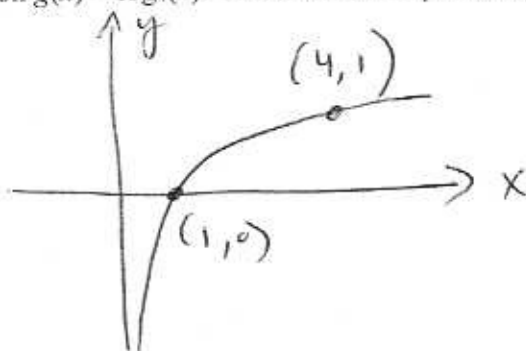
$$S_{100} = \frac{-32 + 562}{2} \cdot 100 = \boxed{26500}$$

$$\begin{array}{r} 99 \\ 6 \\ \hline 594 \\ -32 \\ \hline 562 \\ -32 \\ \hline 530 \end{array} \quad (5 \text{ pts})$$

7a) Graph the function $f(x) = 5 \cdot 2^x$. Label at least 2 points on the graph.



7b) Graph the function $g(x) = \log_4(x)$. Label at least 2 points on the graph.



8)

a) Simplify: $(9)^{5/2}$

$$3^5 = 243$$

b) Simplify and rewrite without a negative exponent: $(x^4)^{-3/8}$

$$x^{-3/2} = \frac{1}{x^{3/2}}$$

$$4 \left(\frac{3}{2} \right)$$

Key

9) Solve and write answers to at least 3 significant figures.

a) $\log_8(x) = -3.22$

$$0.00124$$

b) $10^x = 92.07$

$$1.96$$

c) $\log_3(119.2) = x$

$$4.35$$

10) After you get cancelled, your social media followers drop exponentially. You start with 230,000 followers. Then 10 days after Cancel Day, you have 157,000.

a) Find an exponential model.

b) How many followers do you have 27 days after Cancel Day?

c) When do you have only 10,000 followers?

Hint: Use an exponential model of the form $N(t) = A \cdot e^{kt}$. And solve for k .

$$N(t) = 230000 e^{kt}$$

$$157000 = 230000 e^{k \cdot 10}$$

$$k = -0.03818$$

$$N(t) = 230000 e^{-0.03818t}$$

$$b) \boxed{82,034 \text{ followers}}$$

$$c) 10 = 230 e^{kt}$$

$$\ln\left(\frac{10}{230}\right) = kt$$

$$k$$

$$\boxed{82 \text{ days}}$$

11) You have a bunch of nickels and quarters. There are 209 coins worth \$25.25. How many of each coin do you have?

$$N + Q = 209$$

$$5N + 25Q = 2525$$

$$N + 5Q = 505$$

$$N + Q = 209$$

$$4Q = 296$$

74 quarters
135 nickels

12) Find the coefficient of x^7 in the expansion of $(1+x)^{19}$.

(5 pts)

$$\frac{19!}{12!7!}$$

$$\frac{19 \cdot 18 \cdot 17 \cdot 16 \cdot 15 \cdot 14 \cdot 13}{5040}$$

50388